

Basic theorems of differential calculus and their application

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Annotation. This article gives you some easy ways to solve common. Basic theorems of differential calculus and their application

Key words: vector, inequality, angle, identity

We can often use theorems on derivative functions to solve some problems. These theorems play an important role in checking functions.

Theorem 1 (Fermat theorem).

$f(x)$ function $X \subset R$ given in the package. $x_0 \in X$ for the circumference of the point $U_\delta(x_0) = (x_0 - \delta, x_0 + \delta) \subset X$ ($\delta > 0$) The following conditions must be met:

- 1) $\forall x \in U_\delta(x_0)$ da $f(x) \leq f(x_0)$ ($f(x) \geq f(x_0)$),
- 2) $f'(x_0)$

be available and limited.

Then $f'(x_0) = 0$ is being..

Let's say, $\forall x \in U_\delta(x_0)$ in $f(x) \leq f(x_0)$ let it be Obviously, in this case

$$f(x) - f(x_0) \leq 0$$

will be. Conditionally $f(x)$ function x_0 limited in point $f'(x_0)$ yield. Then

$$f'(x_0) = \lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{x \rightarrow x_0+0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{x \rightarrow x_0-0} \frac{f(x) - f(x_0)}{x - x_0}$$

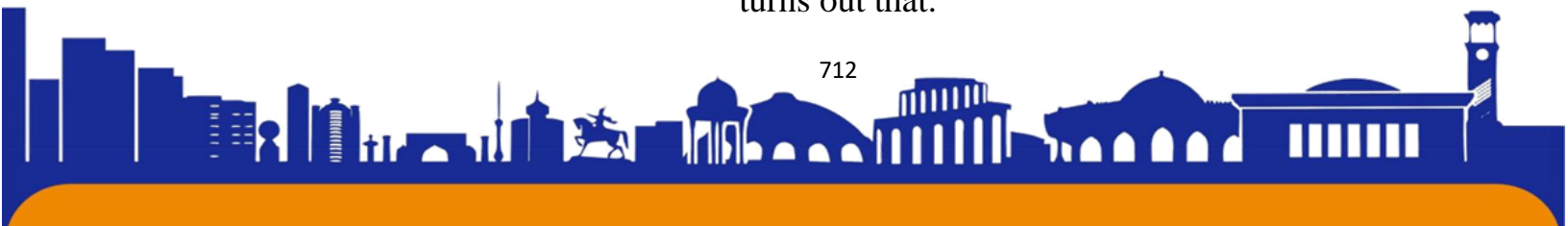
Will be. At the moment, $x > x_0$ will be

$$\frac{f(x) - f(x_0)}{x - x_0} \leq 0 \Rightarrow \lim_{x \rightarrow x_0+0} \frac{f(x) - f(x_0)}{x - x_0} = f'(x_0) \leq 0,$$

$x < x_0$ will be

$$\frac{f(x) - f(x_0)}{x - x_0} \geq 0 \Rightarrow \lim_{x \rightarrow x_0-0} \frac{f(x) - f(x_0)}{x - x_0} = f'(x_0) \geq 0 \text{ from } f'(x_0) = 0 \text{ It}$$

turns out that.





Theorem 2 (Roll theorem). Suppose, $f(x)$ function $[a, b]$ to meet the following conditions:

- 1) $f(x) \in C[a, b]$,
- 2) $\forall x \in (a, b)$ in $f'(x)$ available and limited,
- 3) $f(a) = f(b)$ let it be. $x_0 \in (a, b)$ $f'(x_0) = 0$

Conditionally $f(x) \in C[a, b]$. According to Weierstrass's second theorem $f(x)$ function $[a, b]$ at its maximum and minimum values, c_1, c_2 points ($c_1, c_2 \in [a, b]$) found:

$$f(c_1) = \max\{f(x) \mid x \in [a, b]\},$$

$$f(c_2) = \min\{f(x) \mid x \in [a, b]\}$$

Has been.

If $f(c_1) = f(c_2)$ been, Then $[a, b]$ in $f(x) = const$ is being, $\forall x_0 \in (a, b)$ at $f'(x_0) = 0$.

If $f(c_1) > f(c_2)$ be, that's $f(a) = f(b)$ because $f(x)$ function $f(c_1)$ and $f(c_2)$ to at least one of the values $[a, b]$ the interior of the segment x_0 ($a < x_0 < b$) .reach the point According to the farm theorem $f'(x_0) = 0$ will be. ►

3-theorem (Lagranj by theorem). Suppose, $f(x)$ function $[a, b]$ at will be, fulfill the following conditions:

- 1) $f(x) \in C[a, b]$,
- 2) $\forall x \in (a, b)$ at $f'(x)$ the product is available and limited In that case it is so $c \in (a, b)$ poind found ,

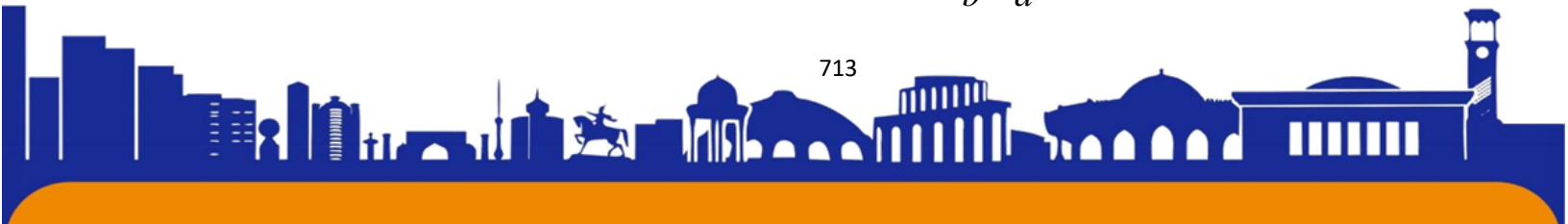
$$f(b) - f(a) = f'(c)(b - a)$$

will be.

$$\text{This } F(x) = f(x) - f(a) - \frac{f(b) - f(a)}{b - a}(x - a) \tag{1}$$

Let's look at the function. This function satisfies all the conditions of the Roll theorem. At the same time, its a product

$$F'(x) = f'(x) - \frac{f(b) - f(a)}{b - a}$$





will be. According to the roll theorem, so c ($c \in (a, b)$) the point is found,

$$F'(c) = 0 \tag{2}$$

Will be.

(1) and (2) from equations

$$f'(c) - \frac{f(b) - f(a)}{b - a} = 0, \text{ that is } f(b) - f(a) = f'(c)(b - a)$$

will occur.

1-result. Let's say, $f(x)$ function (a, b) at $f'(x)$, having a product $\forall x \in (a, b)$ at $f'(x) = 0$ being to. Then $\forall x \in (a, b)$ at $f(x) = const$ will be.

$x, x_0 \in (a, b)$ take, edges x and x_0 in the segment $f(x)$ using Lagrange's theorem on the function $f(x) = f(x_0) = const$ being found. ►

2-result. $f(x)$ and $g(x)$ function (a, b) at $f'(x), g'(x)$, products $\forall x \in (a, b)$ in $f'(x) = g'(x)$ been. Then $\forall x \in (a, b)$ in $f(x) = g(x) + const$ will be.

This is proof of the result $f(x) - g(x)$ by applying result 1 to the function. Theorem 4 (Cauchy Theorem). Let, and let the functions fulfill the following conditions.

- 1) $f(x) \in C[a, b], g(x) \in C[a, b],$
- 2) $\forall x \in (a, b)$ da $f'(x)$ va $g'(x)$ crops are available and limited;
- 3) $\forall x \in (a, b)$ da $g'(x) \neq 0$ will be.

Then $c \in (a, b)$, the point is found

$$\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)}$$

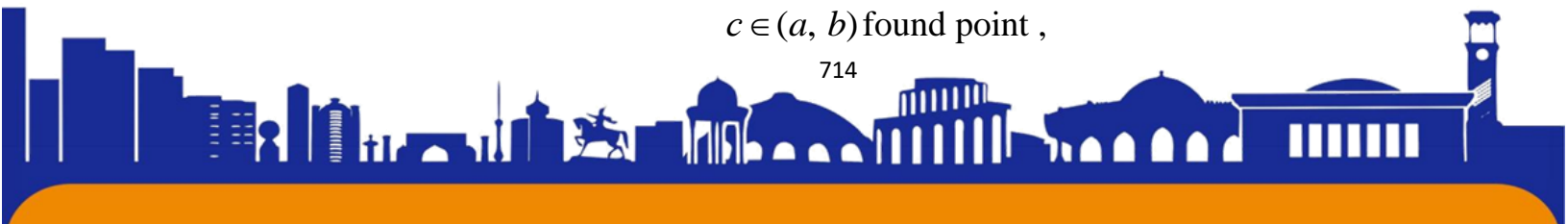
Will be.

First of all $g(b) \neq g(a)$ We emphasize that because $g(b) = g(a)$ if so, then according to Roll's theorem $c \in (a, b)$ the point would be found $g'(c) = 0$ would be This is contrary to condition

3) The following

$$\Phi(x) = f(x) - f(a) - \frac{f(b) - f(a)}{g(b) - g(a)} [g(x) - g(a)] \quad (x \in [a, b])$$

$c \in (a, b)$ found point ,





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$\Phi'(c) = 0$ will be (3)

$$\Phi'(x) = f'(x) - \frac{f(b) - f(a)}{g(b) - g(a)} g'(x) \quad (4)$$

Obviously,

(3) and (4) relationship

$$f'(c) - \frac{f(b) - f(a)}{g(b) - g(a)} g'(c) = 0$$

that is
$$\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)}$$

will occur.

Example 1. $\forall x', x'' \in R$ for $|\sin x' - \sin x''| \leq |x' - x''|$ prove the inequality.

Let's say, $x' < x''$ will be. $f(x) = \sin x$ in $[x', x'']$ We apply Lagrange's theorem. That's it $c \in (x', x'')$ the point is that,

$$|\sin x' - \sin x''| = |\cos c| \cdot (x'' - x')$$

will be. If $\forall t \in R$ at $|\cos t| \leq 1$ Given that, then from the above relationship,

$$|\sin x' - \sin x''| \leq |x' - x''| \quad (\forall x', x'' \in R)$$

Being.

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