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# BETA VA GAMMA FUNKSIYA VA UNING TADBIQLARI

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**Annotatsiya:** Ushbu maqolada Beta hamda Gamma funksiya hamda ularning tadbiqlarini o'rganib chiqamiz.

**Kalit so'zlar:** Beta funksiya, Gamma funksiya, integral, Eyler integrali, I tur Eyler integrali, II tur Eyler integrali.

**Beta funksiya [ I tur Eyler integrali ].**

**1.1-Tarif.**

$$\int_0^1 x^{a-1}(1-x)^{b-1}dx$$

integral *beta finksiy* yoki *I tur Eyler integrali* deb ataladi va *B*(a, b) kabi belgilanadi. De-mak

$$B\,(\mathrm{a,b}) = \int_0^1 x^{a-1}(1-x)^{b-1}dx \quad (a>0, b>0)$$

bo'ladi.

**Gamma funksiya [ II tur Eyler integrali ].**

**1.2-Tarif.**

$$\int_0^{+\infty} x^{a-1}e^{-x}\,dx$$

integral *gamma funksiya* yoki II *tur Eyler integrali* deb ataladi va *Γ* (a) kabi belgilanadi. Demak

$$\Gamma\,(\mathrm{a}) = \int_0^{+\infty} x^{a-1}e^{-x}\,dx \quad (0, +\infty)$$

bo'ladi.

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Biz quyida $B$(a, b) va $\Gamma$ (a) funksiyalar orasidagi bog'lanishni ifodalaydigan formulani keltiramiz.

Ma'lumki, $\Gamma$ (a) funksiya (0, + ∞) da, $B$(a, b) funksiya esa $R^2$ fazodagi $M$ = {(a, b) ∈ ∈ $R^2$ : a ∈ $(0, +\infty)$, b ∈ $(0, +\infty)$ } to'plam berilgan.

**1.1– Teorema.** $\forall\, (a, b) \in M$ uchun

$B$(a, b) = $\frac{\Gamma(a)\cdot\Gamma(b)}{\Gamma(a+b)}$

*fo'rmula o'rinlidir.*

**Isbot.** Ushbu

$$\Gamma(a+b) = \int_0^{+\infty} x^{a+b-1}\cdot e^{-x}dx$$

( a > 0, b > 0) gamma funksiyada o'zgaruvchni quyidagicha almashtiramiz:

x = (1 + $t$) $y$   ($t$ > o).

Natijada quyidagiga ega bo'lamiz:

$$\Gamma(a+b) = \int_0^{+\infty} (1+t)^{a+b-1}\cdot y^{a+b-1}\, e^{-(1+t)y}\cdot(1+t)dy =$$

$$= (1+t)^{a+b}\int_0^{+\infty} y^{a+b-1}\, e^{-(1+t)y}\, dy.$$

Keyingi tenglikdan quyidagini topamiz:

$$\frac{\Gamma(a+b)}{(1+t)^{a+b}} = \int_0^{+\infty} y^{a+b-1}\, e^{-(1+t)y}\, dy$$

Bu tenglikning har ikki tomonini $t^{\alpha-1}$ ga ko'paytirib, natijani $(0, +\infty)$ oraliq bo'yicha intigrallaymiz:

$$\Gamma(a+b)\int_0^{+\infty}\frac{t^{a-1}}{(1+t)^{a+b}}\,dt = \int_0^{+\infty}\left[\int_0^{+\infty} y^{a+b-1}\, e^{-(1+t)y}\, dy\right]t^{a-1}dt.$$

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Agar

$$\int_0^{+\infty} \frac{t^{a-1}}{(1+t)^{a+b}}\,dt = B(a,b) \qquad (1)$$

(1) formulaga ko'ra

$$\Gamma(a+b)\cdot B(a,b) = \int_0^{+\infty}\left[\int_0^{+\infty} y^{a+b-1}\,e^{-(1+t)y}\,dy\right]t^{a-1}dt. \quad (2)$$

bo'ladi. Endi (2) tenglikning o'ng tomonidagi integral $\Gamma(a)\cdot\Gamma(b)$ ga teng bo'lishini isbotlaymiz. Uning uchun, avvalo bu integrallarda integrallash tartibini almashtirish mumkinligini ko'rsatamiz. Buning uchun biz parametirga bog'liq xosmas integrallarni parameter bo'yicha integralash mavzusizdagi (**17.19 – teoremadan**) foydalanamiz.

**17.19-Teorema** shartlari bajarilishini ko'rsatishimiz kerak.

Dastlab $a > 1$, $b > 1$ bo'lgan holni qaraylik.

$a > 1$, $b > 1$ da, yani $\{(a, b) \in R^2 : a \in (1, +\infty), b \in (1, +\infty)\}$ to'plamda integral ostidagi

$$f(t,y) = y^{a+b-1}\,t^{a-1}\,e^{-(1+t)y}$$

funksiya $\forall (t,y) \in \{(t,y) \in R^2 : t \in [0,+\infty), y \in [0,+\infty)\}$ da uzluksiz bo'lib, $f(t,y) = y^{a+b-1}\,t^{a-1}\,e^{-(1+t)y} \geq 0$ bo'ladi.

Ushbu

$$\int_0^{+\infty} f(t,y)\,dt = \int_0^{+\infty} t^{a-1}\,y^{a+b-1}\,e^{-(1+t)y}\,dt$$

integral $\boldsymbol{y}$ o'zgaruvchining $[0, +\infty)$ oraliqdagi uzluksiz fungsiyasi bo'ladi, chunki

$$\int_0^{+\infty} t^{a-1}\,y^{a+b-1}\,e^{-(1+t)y}\,dt = \Gamma(a)\cdot y^{b-1}e^{-y}$$

va nihoyat, yuqoridagi (2) munosabatga ko'ra

$$\int_0^{+\infty}\left[\int_0^{+\infty} t^{a-1}\,y^{a+b-1}\,e^{-(1+t)y}\,dy\right]dt$$

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integral yaqinlashuvchi kelib chiqadi.

U holda **17.19** **Teoremaga** asosan

$$\int_0^{+\infty}\left[\int_0^{+\infty} t^{a-1}\, y^{a+b-1}\, e^{-(1+t)y}\, dt\right] dy$$

integral ham yaqinlashuvchi bo'lib,

$$\int_0^{+\infty}\left[\int_0^{+\infty} t^{a-1}\, y^{a+b-1}\, e^{-(1+t)y}\, dy\right] dt = \int_0^{+\infty}\left[\int_0^{+\infty} t^{a-1}\, y^{a+b-1}\, e^{-(1+t)y}\, dt\right] dy$$

bo'ladi. O'ng tomondagi integralni hisoblasak,

$$\int_0^{+\infty}\left[\int_0^{+\infty} t^{a-1}\, y^{a+b-1}\, e^{-(1+t)y}\, dy\right] dt =$$

$$= \int_0^{+\infty} y^{b-1}\, e^{-y}\, \Gamma(a) dy = \Gamma(a)\cdot\Gamma(b). \qquad (3)$$

tenglik kelib chiqadi.

Natijada (2) va (3) munosabatlardan

$$\Gamma\,(\mathrm{a}+\mathrm{b})\cdot B(\mathrm{a},\mathrm{b}) = \Gamma(a)\cdot\Gamma(b)$$

yani

$$B(\mathrm{a},\mathrm{b}) = \frac{\Gamma(a)\cdot\ \Gamma(b)}{\Gamma(a+b)} \qquad (4)$$

bo'lishi kelib chiqadi.

**2.1 – Natija.** $\forall\, a\ \in (0,1)$ uchun

$$\Gamma(a)\cdot\Gamma(1-a) = \frac{\pi}{\sin \mathrm{a}\,\pi} \qquad (6)$$

bo'ladi.

Haqiqattan ham, (4) formula b = 1 – a (0 < a < 1) deyilsa, unda

$$B(\mathrm{a},1-\mathrm{a}) = \frac{\Gamma(a)\cdot\ \Gamma(1-a)}{\Gamma(1)}$$

bo'ladi.

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Xususan *beta funksiyada* b = 1 – a (0 < a < 1) bo'lganda

$$B\,(\mathrm{a}, 1 - \mathrm{a}) = \int_0^{+\infty} \frac{t^{a-1}}{1+t}\,dt = \frac{\pi}{\sin \mathrm{a}\,\pi} \qquad (7)$$

(7) munosabat o'rinli bo'ladi. (7) va $\Gamma(1) = 1$ munosabatlarga muofiq

$\Gamma(a) \cdot \Gamma(1-a) = \frac{\pi}{\sin \mathrm{a}\,\pi}$ (0 < a < 1).

(6) formula ***keltirish formulasi*** deb ataladi.

**Misol.** Ushbu

$$\int_0^{\infty} \frac{\sin(x) \ln x}{x}\,dx$$

integralni hisoblamg.

Yechish. Integralni hisoblaymiz .

$$\int_0^{\infty} \frac{\sin(x) \ln x}{x}\,dx \qquad [1]$$

[1] integralni **I** bilan belgilab olamiz

$$\mathrm{I} = \int_0^{\infty} \frac{\sin(x) \ln x}{x}\,dx \quad [2].$$

Endi **S** parametirga bog'liq xosmas integral olamiz

$$\mathrm{I(s)} = \int_0^{\infty} \frac{\sin(x)}{x^s}\,dx \quad [3]$$

bu integral S ≥ 1 o'suvchi bo'ladi. [3] integralni S bo'yicha ikkala tomoni ham differen- sialaymiz

$$\frac{d}{ds}\mathrm{I(s)} = \frac{d}{ds}\int_0^{\infty} \frac{\sin(x)}{x^s}\,dx \quad [4]$$

[4] tenglama hosil bo'ladi. [4] tenglamani yechsak

$$\frac{d}{ds}\mathrm{I(s)} = \frac{d}{ds}\int_0^{\infty} \frac{\sin(x)}{x^s}\,dx = \int_0^{\infty} \frac{2}{2s}\frac{\sin x}{x^s}\,dx$$

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teng bo'ladi. $\frac{2}{2s}\,x^{-s} = -x^{-s}\ln x$ ga tenglasak

$$\mathrm{I}'(\mathrm{s}) = -\int_0^{\infty} \frac{\sin x}{x^s} \ln x \; dx \quad [5]$$

[5] tenglamaga kelamiz. Bunda $-\mathrm{I}'$ (s = 1) = I yani [5] tenglama [2] tenglamaga teng bo'ladi.

Endi [1] integralni yechish uchun [3] integraldan foydalanamiz

$$\mathrm{I}(\mathrm{s}) = \int_0^{\infty} \frac{\sin(x)}{x^s}\, dx = \int_0^{\infty} \sin x \frac{1}{x^s}\, dx \quad [6]$$

bo'ladi. Gamma fungsiyadan foydalanib

$$I_1 = \int_0^{\infty} t^{s-1} e^{-tx} dt \quad [7]$$

[7] da $tx = u$ deb $t$ bo'ycha differensialasak va $\mathrm{dt} = \frac{1}{x}\,du$ ni topsak, [7] tenglama

$$I_1 = \int_0^{\infty} \frac{u^{s-1}}{x^{s-1}} \frac{du}{x} = \frac{1}{x^s} \int_0^{\infty} u^{s-1} e^{-u} du$$

$$I_1 = \frac{1}{x^s} \int_0^{\infty} u^{s-1} e^{-u} du \quad [8]$$

[8] ko'rnishida bo'ladi.Bunda [8] ni chap tomonidagi integral $\Gamma(s)$ teng.

$$\Gamma(s) = \int_0^{\infty} u^{s-1} e^{-u} du$$

[8] ni chap tomonidagi integral o'rniga $\Gamma(s)$ ni qo'ysak

$$I_1 = \frac{1}{x^s}\,\Gamma(s) \quad [9]$$

[9] ga teng bo'ladi. Bunda

$$\frac{1}{x^s} = \frac{1}{\Gamma(s)} \int_0^{\infty} t^{s-1} e^{-tx} dt$$

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belgilab, [6] ga quysak

$$\mathrm{I(s)} = \int_0^\infty \sin x \left\{ \frac{1}{\Gamma(s)} \int_0^\infty t^{s-1} e^{-tx} dt \right\} dx \quad [10]$$

teng.

$$\mathrm{I(s)} = \int_0^\infty \sin x \left\{ \frac{1}{\Gamma(s)} \int_0^\infty t^{s-1} e^{-tx} dt \right\} dx = \frac{1}{\Gamma(s)} \int_0^\infty \int_0^\infty (\sin x) t^{s-1} e^{-tx} dt dx$$

$$\mathrm{I(s)} = \frac{1}{\Gamma(s)} \int_0^\infty \int_0^\infty (\sin x) t^{s-1} e^{-tx} dx dt = \frac{1}{\Gamma(s)} \int_0^\infty t^{s-1} \left\{ \int_0^\infty (\sin x)\, e^{-tx} dx \right\} dt$$

bo’laklab integralasak

$$\int_0^\infty (\sin x)\, e^{-tx} dx = \frac{1}{1+t^2}$$

ga teng bo'ladi.

$$\mathrm{I(s)} = \frac{1}{\Gamma(s)} \int_0^\infty \frac{t^{s-1}}{1+t^2} dt$$

bunda $t^2 = y \;=>\; dt = \frac{1}{2} y^{-1/2} dy$ belgilab integralga quysak

$$\mathrm{I(s)} = \frac{1}{2\Gamma(s)} \int_0^\infty \frac{y^{\frac{s-1}{2}} y^{-1/2}}{1+y} dy = \frac{1}{2\Gamma(s)} \int_0^\infty \frac{y^{\frac{s}{2}-1}}{1+y} dy$$

$\frac{s}{2} = z$ deb

$$\int_0^\infty \frac{y^{z-1}}{1+y} dy = \Gamma(z)\Gamma(1-z) = \frac{\pi}{\sin z\pi}$$

$$\mathrm{I(s)} = \frac{1}{2\Gamma(s)} \frac{\pi}{\sin z\pi} \quad [11]$$

[11] ga teng bo’ladi. Bunda $z = \frac{s}{2}$ [11] ga oborib qo’yamiz

$$\mathrm{I(s)} = \frac{1}{2\Gamma(s)} \frac{\pi}{\sin \frac{s\pi}{2}} \quad [12]$$

teng bo’ladi.

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Agar $I = -I(s=1)$ bo'lsa, [12] tenglamadan s bo'yicha hosila olsak

$$\mathrm{I}'(s) = \frac{\pi}{2}\left\{\frac{-\Gamma'(s)}{\Gamma^2(s)}\frac{1}{\sin\frac{s\pi}{2}} - \frac{1}{2\Gamma(s)}\cdot\frac{-\cos\frac{s\pi}{2}}{(\sin\frac{s\pi}{2})^2}\right\}$$

$$\mathrm{I}'(1) = \frac{\pi}{2}\left\{-\Gamma'(1)\frac{1}{\sin\frac{\pi}{2}} - \frac{1}{2}\cdot\frac{-\cos\frac{\pi}{2}}{(\sin\frac{\pi}{2})^2}\right\}$$

$$\mathrm{I}'(1) = -\frac{\pi}{2}\Gamma'(1)$$

bunda $-\Gamma'(1) = \gamma$ ga teng

$$\mathrm{I}'(1) = -\frac{\pi}{2}\Gamma'(1) = \frac{\pi\gamma}{2}$$

$$\int_0^\infty \frac{\sin(x)\ln x}{x}\,dx = \frac{\pi\gamma}{2}.$$

teng bo'ladi. Misol yechildi.

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