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BIR ARGUMENTNING FUNKSIYALARI QATNASHGAN BA'ZI TRIGONOMETRIK TENGLAMALARNING YECHIMLARI

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Annotatsiya

Ushbu maqolada ayrim trigonometrik tenglamarni universal almashtirish formulalari yordamida yechish keltiriligan.

Hamma trigonometrik funksiyalarni sinx va cosx lar orqali ifodalash mumkin bo'lgani uchun, bir argumentning trigonometrik funksiyalariga nisbatan ratsional tenglamalarni

$$R(\sin x, \cos x) = 0$$

ko'rinishida yozish mumkin (bunda R ifoda sinx va cosx ga nisbatan ratsional funksiya).

$R(\sin x, \cos x) = 0$ ko'rinishdagi tenglamalarni $tg\frac{x}{2}$ ga nisbatan ratsional bo'lgan tenglamalarga keltirib yechish mumkin.

1-misol. $3\sin x + \sqrt{3}\cos x = 3$ tenglamani yeching.

Yechish. Tenglamaning aniqlanish sohasi: (-∞,+∞). sinx va cosx larni universal almashtirish formulalaridan foydalanib almashtirsak,

$$3 \cdot \frac{2tg\frac{x}{2}}{1 - tg^2\frac{x}{2}} + \sqrt{3} \cdot \frac{1 - tg^2\frac{x}{2}}{1 + tg^2\frac{x}{2}} = 3$$

tenglamadan

$$\left(\sqrt{3} + 1\right)tg^2\frac{x}{2} - 2\sqrt{3}tg\frac{x}{2} + \left(\sqrt{3} - 1\right) = 0$$

Tenglamani hosil qilamiz. Buni yechsak,

$$tg\frac{x_1}{2} = 1 \text{ dan } \frac{x_1}{2} = \frac{\pi}{4} + k\pi \text{ yoki } x_1 = \frac{\pi}{2} + 2k\pi.$$

$$tg\frac{x_2}{2}=\frac{\sqrt{3}-1}{\sqrt{3}+1} \text{ dan } \frac{x_2}{2}=arctg\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)+k\pi$$

yoki

$$x_2=2arctg\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)+2k\pi .$$

Lekin almashtirish natijasida berilgan tenglamaning aniqlanish sohasi x=π(2k+1) ga kichraygan edi. x=π(2k+1) qiymatlarning berilgan tenglamaning yechimlari bo‘lish-bo‘lmasligini tekshiramiz. Buning uchun sinx va cosx larning davri 2π bo‘lgani uchun x= π ni tekshirsak kifoya, ya’ni

$$3\sin\pi+\sqrt{3}\cos\pi=3 \text{ yoki } -\sqrt{3}\neq 3.$$

Demak, x=π(2k+1) tenglamaning yechimi bo‘la olmaydi.

$$\text{Javob: } x_1=\frac{\pi}{2}(4k+1); \quad x_2=2arctg\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)+2k\pi.$$

2-misol. $\sin^2 x+0.5\sin 2x+2\cos x=2$ tenglamani yeching.

Yechish. Tenglamaning aniqlanish sohasi: (-∞,+∞). sinx va cosx larni $tg\frac{x}{2}$ ga almashtirib,

$$\left(\frac{2tg\frac{x}{2}}{1-tg^2\frac{x}{2}}\right)^2+0.5\cdot 2\left(\frac{2tg\frac{x}{2}}{1-tg^2\frac{x}{2}}\right)\cdot\left(\frac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}}\right)+2\left(\frac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}}\right)=2$$

tenglamadan

$tg^3\frac{x}{2}-tg^2\frac{x}{2}-tg\frac{x}{2}=0$ yoki $tg\frac{x}{2}=0$ va $tg^2\frac{x}{2}-tg\frac{x}{2}-1=0$ tenglamalarni hosil qilamiz.

Bu tenglamalarni yechib,

$$x_1=2\pi k \quad \text{va} \quad x_{2,3}=2arctg\left(\frac{1\pm\sqrt{5}}{2}\right)+2k\pi$$

larni topamiz. Lekin almashtirish natijasida berilgan tenglamaning aniqlanish sohasi x=π(2k+1) ga kichraygan edi. Tenglamani x= π da tekshiramiz, ya’ni

$$\sin^2\pi + 0.5\sin 2\pi + 2\cos\pi = 2 \text{ yoki} -2 \neq 2.$$

Demak, x=π(2k+1) tenglamaning yechimlari emas.

$$\text{Javob: } x_1 = 2\pi k, \quad x_{2,3} = 2arctg\left(\frac{1\pm\sqrt{5}}{2}\right) + 2k\pi.$$

sinx, cosx tgx va ctgx larni $tg\frac{x}{2}$ ga almashtirish universal trigonometrik almashtirish deyiladi. Bu almashtirish faqat tenglamalarga boshqa almashtirishlarni ishlatish mumkin bo'lmagan hollardagina ishlatiladi, chunki natijada yuqori darajali tenglamalar hosil bo'lishi mumkin.

$a\sin x + b\cos x = c$ ko'rinishidagi tenglamalarni yechish.

$a\sin x + b\cos x = c$ ko'rinishidagi tenglama, (a, b, c lar o'zgarmas sonlar) $R(\sin x, \cos x) = 0$ tenglamaning hususiy holi bo'lib, uni universal almashtirish yordami bilan yechish mumkin. Bu tenglama quyidagicha ham yechiladi.

Tenglamani

$$\frac{a}{\sqrt{a^2+b^2}}\sin x + \frac{b}{\sqrt{a^2+b^2}}\cos x = \frac{c}{\sqrt{a^2+b^2}}$$

ko'rinishida yozib,

$$\cos\varphi = \frac{a}{\sqrt{a^2+b^2}}, \qquad \sin\varphi = \frac{b}{\sqrt{a^2+b^2}}$$

deb olsak,

$$\cos\varphi\sin x + \sin\varphi\cos x = \frac{c}{\sqrt{a^2+b^2}}$$

ni hosil qilamiz. Bundan

$$\sin(\varphi + x) = \frac{c}{\sqrt{a^2+b^2}}.$$

U holda $\left|\frac{c}{\sqrt{a^2+b^2}}\right| \leq 1$ bo'lsa, tenglamaning yechimlari

$$x + \varphi = (-1)^n \arcsin\frac{c}{\sqrt{a^2+b^2}} + \pi k \text{ yoki}$$

$x = (-1)^n \arcsin\frac{c}{\sqrt{a^2+b^2}} - \varphi + \pi k$ bo'lib $\left|\frac{c}{\sqrt{a^2+b^2}}\right| > 1$ bo'lsa, tenglama yechimga ega emas.

3-misol. $\sin x+\sqrt{3}\cos x=\sqrt{2}$ tenglamani yeching.

Yechish. tenglamaning ikkala tomonini $\sqrt{1^2+\sqrt{3}^2}=2$ ga bo'lib,

$$\frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x=\frac{\sqrt{2}}{2}$$

ni hosil qilamiz.

$$\cos\frac{\pi}{3}=\frac{1}{2} \quad \text{va} \quad \sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$$

Bo'lgani uchun, yuqoridagi tenglama

$$\cos\frac{\pi}{3}\sin x+\sin\frac{\pi}{3}\cos x=\frac{\sqrt{2}}{2}$$

yoki

$$\sin\left(\frac{\pi}{3}+x\right)=\frac{\sqrt{2}}{2}$$

ko'rinishga keladi. Bu yerdan

$$x+\frac{\pi}{3}=(-1)^n\frac{\pi}{4}+\pi k$$

yoki

$$x=(-1)^n\frac{\pi}{4}-\frac{\pi}{3}+\pi k.$$

Foydalanilgan adabiyotlar.

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