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YORDAN ALGEBRALARI VA ULAR USTIDA UMUMIYLASHTIRILGAN DIFFERENSIALLASHLAR HAQIDA

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Annotatsiya. Ushbu maqolada Yordan algebralarida umumiylashtirilgan differensiallash tushunchasi kiritilgan va uning umumiy xossalari o'rganilgan. Xususan, Yordan algebralarida berilgan umumiylashtirilgan differensiallashlar yordamida invariantla rqurilgan. Shuningdek,Yordan algebralarida(α,β,γ)-differensiallashlar to'plamining α,β,γ parametrlarining ayrim qiymatlari bo'yicha tavsifi keltirilgan.

Kalitso'zlar. Vektor fazo, Yordan algebrasi, chiziqli akslantirish, differensiallash, umumiylashtirilgan differensiallash.

*1-Ta'rif***.** Yordan algebrasi J – bu quyidagi ayniyatlarni qanoatlantiruvchi

$$J\times J\to J$$

Bu chiziqli akslantirish berilgan F maydonidagi V vektor fazosidir

$$a\cdot b=b\cdot a, a,b\in J,$$

$$(a^2\cdot b)\cdot a=a^2\cdot(b\cdot a), a,b\in J.$$

Yordan algebralarining (α,β,γ)-differensiallashlari Ta'rifini quyidagicha yozish mumkin.

2-Ta'rif. Aytaylik $(J,\cdot)$ – Yordan algerasi bo'lsin. Chiziqli operator $d\in End(J)$ J Yordan algebrasining (α,β,γ) – differensiyalashi deb ataladi, agar shunday $\alpha,\beta,\gamma\in\mathbf{C}$ mavjud bo'lib, barcha $x,y\in J$ elementlar uchun quyidagi shart bajarilsa

$$\alpha d(x\cdot y) = \beta(d(x)\cdot y) + \gamma(x\cdot d(y)).$$

J Yordan algebrasining barcha (α,β,γ) – differensiallashlari to'plamini $Der_{(\alpha,\beta,\gamma)}(J)$ orqali belgilaylik. Ushbu to'plam quyidagicha aniqlanadi

$$Der_{(\alpha,\beta,\gamma)}(J) = \{d\in End(J):\ \alpha d(x\cdot y) = \beta(d(x)\cdot y) + \gamma(x\cdot d(y)), x,y\in J\}.$$

Bu yerda $Der_{(\alpha,\beta,\gamma)}(J)$ to'plam $End(J)$ ning vektor qism fazosi bo'ladi.

Teorema. J – birlik elementli Yordan algebrasi va $\alpha,\beta,\gamma\in\mathbf{F}$ bo'lsin. U holda $Der_{(\alpha,\beta,\gamma)}(J)$ vektor fazo quyidagi ko'rinishlarga ega bo'ladi.

1. $Der_{(1,1,1)}(J) = Der(J)$, bu yerda $Der(J)$ – bu J Yordan algebrasining barcha differensiallashlari Li algebrasi;

2. $Der_{(1,1,0)}(J)\subseteq End(L)$ va $Der_{(1,1,0)}(J)$ to'plamni J Yordan algebrasining barcha idempotent elementlari to'plami $Id(J)$ bilan

$$p\in Id(J), \qquad d(x) = p\cdot x,\ \ x\in J$$

orqali aynan tenglashtirish mumkin.

3. $Der_{(1,1,-1)}(J)\subseteq Der_{(1,0,0)}(J)\equiv 0$.

4. $Der_{(0,1,1)}(J)\equiv 0$.

5. $Der_{(0,1,-1)}(J)$ to'plamni $Z_o(J) = \{a\in J \mid (b\cdot a)\cdot c = b\cdot(a\cdot c),\ \ b,c\in J\}$ to'plam bilan

$$a\in Z_o(J),\ \ d(x) = a\cdot x,\ \ x\in J.$$

orqali aynan tenglashtirish mumkin.

6. $Der_{(0,1,0)}(J)\equiv 0$;

7. Agar $\delta\neq 1$ bo'lsa, u holda $Der_{(\delta,1,0)}(J)\equiv 0$.

Teorema isboti. $d(x\cdot y) = d(x)\cdot y, x, y \in J$ tenglikdan $d(x) = d(e)\cdot x,\ x \in J$ tenglikka ega bo‘lamiz. U holda, J Yordan algebrasi kommutativ bo‘lgani uchun

$$d(x\cdot y) = d(e)\cdot(x\cdot y) = d(x)\cdot y = d(y)\cdot x, \quad x, y \in J$$

va

$$d(x\cdot y) = d(x\cdot e)\cdot y = (d(e)\cdot x)\cdot y = (d(e)\cdot y)\cdot x, \quad x, y \in J.$$

Bundan $d(e)$ – bu markaziy elementdir. Shu bilan birga

$$d(e) = d(e\cdot e) = d(e)\cdot d(e),$$

ya’ni $d(e)$ element idempotent elementdir. Aksincha, J Yordan algebrasining har qanday markaziy idempotenti p uchun

$$d(x) = p\cdot x,\ \ x \in J,$$

akslantirish $Der_{(1,1,0)}(J)$ to‘plamga tegishli va shu yo‘l bilan $Der_{(1,1,0)}(J)$ to‘plamni J Yordan algebrasining barcha idempotent elementlari to‘plami bilan aynan tenglashtirish mumkin.

Bundan esa 1 va 2 ko‘rinishlarning bajarilishi ravshandir.

3. Ta’rifga ko‘ra $Der_{(1,1,-1)}(J) = \{d \in End(J) \mid d(x\cdot y) = d(x)\cdot y - x\cdot d(y),$ $x, y \in J\}$. Quyidagi tengliklar o‘rinli

$$d(x) = d(e)\cdot x - d(x), \qquad x, y \in J,$$

$$d(x\cdot y) = \big(d(e)\cdot x - d(x)\big)\cdot y - x\cdot\big(d(e)\cdot y - d(y)\big) =$$
$$(d(e)\cdot x)\cdot y - d(x)\cdot y - x\cdot(d(e)\cdot y) + x\cdot d(y) =$$
$$(d(e)\cdot x)\cdot y - x\cdot(d(e)\cdot y) - d(x\cdot y), \qquad x, y \in J,$$
$$d(e) = d(e\cdot e) = d(e)\cdot e - e\cdot d(e) = 0.$$

Bundan

$$d(x\cdot y) = -d(x\cdot y) = 0,\ \ x, y \in J$$

va

$$d(e\cdot x) = -d(e\cdot x) = d(x) = 0,\ \ x \in J.$$

Oxirgi tenglikdan ko'rinadiki $Der_{(1,1,-1)}(J) \equiv 0$.

4. Ta'rifga ko'ra, $Der_{(0,1,1)}(J)$ to'plam quyidagicha aniqlanadi

$$Der_{(0,1,1)}(J) = \{d \in End(J) \mid d(x) \cdot y = -x \cdot d(y), x, y \in J\}.$$

U holda $d(e) \cdot x = -e \cdot d(x) = -d(x),\ x \in J$, $d(e) \cdot e = -e \cdot d(e)$, $d(e) = 0$. Bundan

$$d(x) = e \cdot d(x) = -d(e) \cdot x = 0, \qquad x \in J.$$

Demak, $Der_{(0,1,1)}(J) \equiv 0$.

5. Yuqoridagidek, $Der_{(0,1,-1)}(J) = \{d \in End(J) \mid d(x) \cdot y = x \cdot d(y), x, y \in J\}$ to'plam uchun quyidagi tengliklarni hosil qilish mumkin

$$d(e) \cdot x = e \cdot d(x) = d(x),\ \ x \in J,$$

$$d(x) \cdot y = (d(e) \cdot x) \cdot y = \big(x \cdot d(e)\big) \cdot y = x \cdot d(y) = x \cdot (d(e) \cdot y), x, y \in J.$$

Endi

$$Z_o(J) = \{a \in J \mid (b \cdot a) \cdot c = b \cdot (a \cdot c),\ \ b, c \in J\}$$

to'plamdan ixtiyoriy $a \in Z_o(J)$ element olamiz. Quyidagi

$$d(x) = a \cdot x,\ \ x \in J.$$

akslantirish $Der_{(0,1,-1)}(J)$ to'plamga tegishli ekanini ko'rsatamiz. Haqiqatan ham,

$$d(x) \cdot y = (a \cdot x) \cdot y = (x \cdot a) \cdot y = x \cdot (a \cdot y) = x \cdot d(y),\ \ x, y \in J.$$

6. Ta'rifga ko'ra, $Der_{(0,1,0)}(J)$ to'plam quyidagicha aniqlanadi

$$Der_{(0,1,0)}(J) = \{d \in End(J) \mid d(x) \cdot y = 0, x, y \in J\}.$$

Quyidagi tengliklar o'rinli

$$d(x) = d(x) \cdot e = 0, \qquad d(e) = d(e) \cdot e = 0, \qquad x \in J.$$

Bundan $Der_{(0,1,0)}(J) \equiv 0$.

7. Nihoyat, oxirgi

$$Der_{(\delta,1,0)}(J) = \{d \in End(J) \mid \delta d(x \cdot y) = d(x) \cdot y,\ \ x, y \in J\}$$

to'plam quyidagi tengliklarni hosil qilish mumkin

$$\delta d(x) = \delta d(x \cdot e) = d(x) \cdot e = d(x), \quad x \in J$$

$$\delta d(x) = d(x), \qquad (\delta - 1)d(x) = 0, \delta \neq 1, d(x) = 0 \quad x \in J.$$

Chunki

$$0 = (\delta - 1)^{-1} 0 = (\delta - 1)^{-1}(\delta - 1)d(x) = d(x).$$

Demak, $Der_{(\delta,1,0)}(J) \equiv 0$.

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