Ko'p o'zgaruvchili funksiyalarning hususiy hosilalari va differensiallashlar haqida

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Annotatsiya.

Ushbu maqolada ayrim masalalarning hususiy hosilalari va differensiallarini yechilish usullari keltirilgan.

Аннотация

В статье представлены методы решения специальных производных и дифференциалов некоторых задач.

Annotation

The article presents methods for solving special derivatives and differentials of some problems

Kalit so'zlar. Xususiy hosila, differensiallanuvchi funksiya

$f(x)=f(x_1,x_2,...,x_m)$ funksiya ochiq $M\left(M\subset R^m\right)$ to'plamda berilgan bo'lsin. Bu to'plamda $x^0=\left(x_1^0,x_2^0,...,x_m^0\right)$nuqta olib, uning birinchi koordinatasi x_1^0 ga shunday $\Delta x_1\left(\Delta x_1>0,\Delta x_1<0\right)$ orttirma beraylikki, $\left(x_1^0+\Delta x_1,x_2^0,...,x_m^0\right)\in M$ bo'lsin. Natijada $f(x_1,x_2,...,x_m)$funksiya ham $\left(x_1^0,x_2^0,...,x_m^0\right)$ nuqtada x_1 o'zgaruvchisi bo'yicha

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$$\Delta_{x_3} f = f\left(x_1^0 + \Delta x_1, x_2^0, \ldots, x_m^0\right) - f\left(x_1^0, x_2^0, \ldots, x_m^0\right)$$

xususiy orttirmaga ega bo'ladi.

Ushbu

$$\frac{\Delta_{x_1} f}{\Delta x_1} = \frac{f\left(x_1^0 + \Delta x_1, x_2^0, \ldots, x_m^0\right) - f\left(x_1^0, x_2^0, \ldots, x_m^0\right)}{\Delta x_1} \qquad (1.1)$$

nisbatni qaraylik. Ravshanki, bu nisbat Δx_1 ning funksiyasi bo'lib, u Δx_1 ning noldan farqli qiymatlarida aniqlangan.

1.1 - ta'rif. Agar $\Delta x_1 \to 0$ da (1.1) nisbatning limiti

$$\lim_{\Delta x_1 \to 0} \frac{\Delta_{x_1} f}{\Delta x_1} = \lim_{\Delta x_1 \to 0} \frac{f\left(x_1^0 + \Delta x_1, x_2^0, \ldots, x_m^0\right) - f\left(x_1^0, x_2^0, \ldots, x_m^0\right)}{\Delta x_1}$$

mavjud va chekli bo'lsa, bu limit $f\left(x_1, x_2, \ldots, x_m\right)$ funksiyaning $\left(x_1^0, x_2^0, \ldots, x_m^0\right)$ nuqtadagi x_1 o'zgaruvchisi bo'yichaxususiy hosilasi deb ataladi va

$$\frac{\partial f\left(x_1^0, x_2^0, \ldots, x_m^0\right)}{\partial x_1}, \frac{\partial f}{\partial x_1}, f'_{x_1}\left(x_1^0, x_2^0, \ldots, x_m^0\right), f'_{x_1}$$

belgilarning biri bilan belgilanadi. Demak,

$$f'_{x_1}\left(x^0\right) = \frac{\partial f\left(x^0\right)}{\partial x_1} = \lim_{\Delta x \to 0} \frac{\Delta_{x_1} f}{\Delta x_1}.$$

Agar $x_1^0 + \Delta x_1 = x_1$ deb olsak, unda $\Delta x_1 = x_1 - x_1^0$ va $\Delta x_1 \to 0$ da $x_1 \to x_1^0$ bo'lib, natijada

$$\lim_{\Delta x_1 \to 0} = \frac{\Delta_{x_1} f}{\Delta x_1} = \lim_{\Delta x_1 \to 0} \frac{f\left(x_1^0 + \Delta x_1, x_2^0, \ldots, x_m^0\right) - f\left(x_1^0, x_2^0, \ldots, x_m^0\right)}{\Delta x_1} =$$

$$= \lim_{x_1 \to x_1^0} \frac{f\left(x_1, x_2^0, \ldots, x_m^0\right) - f\left(x_1^0, x_2^0, \ldots, x_m^0\right)}{x_1 - x_1^0}$$

bo'ladi. Demak, $f(x_1, x_2, ..., x_m)$ funksiyaning $(x_1^0, x_2^0, ..., x_m^0)$ nuqtadagi x_1 o'zgaruvchisi bo'yichaxususiy hosilasini ushbu

$$\frac{f(x_1, x_2^0, ..., x_m^0) - f(x_1^0, x_2^0, ..., x_m^0)}{x_1 - x_1^0}$$

nisbatning $x_1 \to x_1^0$ dagi limiti sifatida ta'riflash mumkin.

Xuddi shunga o'xshash $f(x_1, x_2^0, ..., x_m^0)$ funksiyaning boshqa o'zgaruvchilari bo'yicha xususiy hosilalari ta'riflanadi:

$$\frac{\partial f}{\partial x_2} = \lim_{\Delta x_2 \to 0} \frac{f(x_1^0, x_2^0 + \Delta x_2, x_3^0, ..., x_m^0) - f(x_1^0, x_2^0, ..., x_m^0)}{\Delta x_2}$$

....

$$\frac{\partial f}{\partial x_m} = \lim_{\Delta x_m \to 0} \frac{f(x_1^0, x_2^0, ..., x_{m-1}^0, x_m^0 + x_m) - f(x_1^0, x_2^0, ..., x_m^0)}{\Delta x_m}$$

Demak, ko'p o'zgaruvchili $f(x_1, x_2^0, ..., x_m^0)$ funksiyaning biror $(x_1^0, x_2^0, ..., x_m^0)$ nuqtada $x_k = (k = 1, 2, ..., m)$ o'zgaruvchisi bo'yicha xususiy hosilasini ta'riflashda bu funksiyaning $x_k = (k = 1, 2, ..., m)$ o'zgaruvchidan boshqa barchao'zgaruvchilari o'zgarmas deb hisoblanar ekan. Shunday qilib, $x_k = (k = 1, 2, ..., m)$ funksiyaning xususiy hosilalari $f'_{x_1}, f'_{x_2}, ..., f'_{x_m}$ bir o'zgaruvchili funksiya hosilasi kabi ekanligini ko'ramiz. Demak, ko'p o'zgaruvchili funksiyalarning xususiy hosilalarini hisoblashda bir o'zgaruvchili funksiyaning hosilasini hisoblashdagi ma'lum bo'lgan qoida va jadvallardan to'liq foydalanish mumkin.

Misollar. 1. $f(x_1, x_2) = \sqrt{x_1^2 + x_2^2}$ bo'lsin. Bu funksiyaning $\forall (x_1, x_2) \in R^2$ nuqtadagi xususiy hosilalari

$$\frac{\partial f}{\partial x_1} = \frac{x_1}{\sqrt{x_1^2 + x_2^2}}, \quad \frac{\partial f}{\partial x_1} = \frac{x_2}{\sqrt{x_1^2 + x_2^2}}$$

bo'ladi.

2. $f(x_1,x_2)=\frac{1}{\sqrt{x_2}}e^{-\frac{x_1+x_2}{2}}$ funksiyaning $\forall(x_1,x_2)\in R^2$ $(x_2>0)$ nuqtadagi xususiy hosilalarini hisoblaymiz:

$$\frac{\partial f}{\partial x_1}=\frac{\partial}{\partial x_1}\left(\frac{1}{\sqrt{x_2}}e^{-\frac{x_1+x_2}{2}}\right)=-\frac{1}{2\sqrt{x_2}}e^{-\frac{x_1+x_2}{2}},$$

$$\frac{\partial f}{\partial x_2}=\frac{\partial}{\partial x_2}\left(\frac{1}{\sqrt{x_2}}e^{-\frac{x_1+x_2}{2}}\right)=-\frac{1}{2\sqrt{x_2^3}}e^{-\frac{x_1+x_2}{2}}-$$

$$-\frac{1}{2\sqrt{x_2}}e^{-\frac{x_1+x_2}{2}}=-\frac{1}{2\sqrt{x_2}}e^{-\frac{x_1+x_2}{2}}\left(1+\frac{1}{x_2}\right).$$

3. Ushbu

$$f(x_1,x_2)=\begin{cases}\frac{2x_1x_2}{x_1^2+x_2^2}, & agar\ (x_1,x_2)\neq(0,0)\ bo'lsa,\\ 0, & agar\ (x_1,x_2)=(0,0)\ bo'lsa\end{cases}$$

funksiyaning xususiy hosilalarini toping.

Aytaylik, $(x_1,x_2)\neq(0,0)$ bo'lsin. U holda

$$\frac{\partial f}{\partial_1}=\frac{\partial}{\partial x_1}\left(\frac{2x_1x_2}{x_1^2+x_2^2}\right)=\frac{2x_2\left(x_1^2+x_2^2\right)-2x_1x_22x_1}{\left(x_1^2+x_2^2\right)^2}=\frac{2x_2\left(x_2^2-x_1^2\right)}{\left(x_1^2+x_2^2\right)^2},$$

$$\frac{\partial f}{\partial_1}=\frac{\partial}{\partial x_1}\left(\frac{2x_1x_2}{x_1^2+x_2^2}\right)=\frac{2x_2\left(x_1^2+x_2^2\right)-2x_1x_22x_2}{\left(x_1^2+x_2^2\right)^2}=\frac{2x_2\left(x_1^2-x_2^2\right)}{\left(x_1^2+x_2^2\right)^2}$$

bo'ladi.

Endi $(x_1,x_2)=(0,0)$ bo'lsin. U holda

$$\frac{\partial f(0,0)}{\partial x_1}=\lim_{\Delta x_1\to 0}\frac{f(\Delta x_1,0)-f(0,0)}{\Delta x_1}=0,$$

$$\frac{\partial f(0,0)}{\partial x_2}=\lim_{\Delta x_2\to 0}\frac{f(0,\Delta x_2)-f(0,0)}{\Delta x_2}=0$$

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bo'ladi.

Demak, berilgan $f(x_1, x_2)$ funksiya $\forall (x_1, x_2) \in R^2$ da xususiy hosilalarga ega.

Funksiyaning differensiallanuvchiligi tushunchasi.

$f(x) = f(x_1, x_2, ..., x_m)$ funksiya ochiq $M\left(M \subset R^2\right)$ to'plamda berilgan bo'lsin. Bu to'plamda $\left(x_1^0, x_2^0, ..., x_m^0\right) = x^0$ nuqta bilan birga $\left(x_1^0 + \Delta x_1, x_2^0 + \Delta x_2, ..., x_m^0 + \Delta x_m\right)$ nuqtani olib, berilgan funksiyaning to'la orttirmasi

$$\Delta f\left(x^0\right) = f\left(x_1^0 + \Delta x_1, x_2^0 + \Delta x_2, ..., x_m^0 + \Delta x_m\right) - f\left(x_1^0, x_2^0, ..., x_m^0\right)$$

ni qaraymiz.

Ravshanki, funksiyaning $\Delta f\left(x^0\right)$ orttirmasi argumentlar orttirmalari $\Delta x_1, \Delta x_2, ..., \Delta x_m$ larga bog'lik, bo'lib, ko'pchilik hollarda $\Delta x_1, \Delta x_2, ..., \Delta x_m$ lar bilan Δf orasidagi bog'lanish murakkab bo'ladi. Tabiiyki, bunda $\Delta x_1, \Delta x_2, ..., \Delta x_m$ larga ko'ra Δf ni aniq yoki taqribiy hisoblash qiyinlashadi. Natijada orttirmasi $\Delta x_1, \Delta x_2, ..., \Delta x_m$ orttirmalar bilan soddaroq bog'lanishda bo'lgan funksiyalarni o'rganish masalasi yuzaga keladi.

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