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Ratsional ko'rinishdagi integrallar va ularning differensiallash usullari orqali yechimlarini topish

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Ushbu maqolada

$$\int R\left(x, \sqrt[n]{\frac{ax+b}{cx+d}}\right)dx \ , \quad ad-bc \neq 0,$$

ratsional ko'rinishdagi integrallarni differensiallashlar o'rganilgan.

1^0. $\int R\left(x, \sqrt[n]{\frac{ax+b}{cx+d}}\right)dx$ ko'rinishidagi integrallarni hisoblash. Faraz qilaylik, $R(u,v)$ ikki o`zgaruvchining ratsional funksiyasi bo`lib, a,b,c,d lar haqiqiy sonlar, $n \in N$ bo`lsin.

Ushbu

$$\int R\left(x, \sqrt[n]{\frac{ax+b}{cx+d}}\right)dx \ , \quad ad-bc \neq 0,$$

ko`rinishidagi integrallarni qaraymiz. Bu integral o`zgaruv-chini almashtirish yordamida ratsional funksiyaning integraliga keladi:

$$\int R\left(x,\sqrt[n]{\frac{ax+b}{cx+d}}\right)dx=\left|\begin{array}{l}\sqrt[n]{\dfrac{ax+b}{cx+d}}=t,\ x=\dfrac{b-t^n d}{ct^n-a}\\ dx=\dfrac{(ad-bc)n}{(a-ct^n)^2}t^{n-1}dt\end{array}\right|=$$

$$=\int R\left(\frac{dt^n-b}{a-ct^n},t\right)\cdot\frac{(ad-bc)nt^{n-1}}{(a-ct^n)^2}dt.$$

1-misol. Ushbu

$$\int\sqrt{\frac{1+x}{1-x}}\cdot\frac{1}{1-x}dx$$

integral hisoblansin.

◀Bu integralda

$$t=\sqrt{\frac{1+x}{1-x}}$$

almashtirishni bajaramiz. Unda

$$x=\frac{t^2-1}{t^2+1},\quad dx=\frac{4tdt}{(t^2+1)^2}$$

bo`lib,

$$\int\sqrt{\frac{1+x}{1-x}}\cdot\frac{1}{1-x}dx=2\int\frac{t^2dt}{t^2+1}$$

bo`ladi.

Ravshanki,

$$\int\frac{t^2dt}{t^2+1}=t-\mathrm{arctg}t+C.$$

Demak,

$$\int\sqrt{\frac{1+x}{1-x}}\cdot\frac{1}{1-x}dx=2\sqrt{\frac{1+x}{1-x}}-2\operatorname{arctg}\sqrt{\frac{1+x}{1-x}}+C \blacktriangleright$$

2^0. $\int R\left(x,\sqrt{ax^2+bx+c}\right)dx$ ko'rinishidagi integrallarni hisoblash. Bu integralda a,b,c -haqiqiy sonlar bo'lib, ax^2+bx+c kvadrat uchhad teng ildizlarga ega emas. Qaralayotgan

$$\int R(x,\sqrt{ax^2+bx+c})dx \qquad (1)$$

integral quyidagi uchta almashtirish yordamida ratsional funksiya integraliga keladi.

a) $a>0$ bo'lsin.

(1) integralda ushbu

$$t=\sqrt{a}x+\sqrt{ax^2+bx+c} \qquad (\text{yoki } t=-\sqrt{a}x+\sqrt{ax^2+bx+c})$$

almashtirishni bajaramiz. U holda

$$ax^2+bx+c=t^2-2\sqrt{a}\,xt+ax^2,$$

$$x=\frac{t^2-c}{2\sqrt{a}t+b}, \qquad dx=\frac{2(\sqrt{a}t^2+bt+c\sqrt{a})}{(2\sqrt{a}t+b)^2}dt,$$

$$\sqrt{ax^2+bx+c}=\frac{\sqrt{a}t^2+bt+c\sqrt{a}}{2\sqrt{a}t+b}$$

bo`ladi.

Natijada

$$\int R(x,\sqrt{ax^2+bx+c})dx=$$

$$=\int R\left(\frac{t^2-c}{2\sqrt{a}t+b},\frac{\sqrt{a}\,t^2+bt+c\sqrt{a}}{2\sqrt{a}\,t+b}\right)\cdot\frac{2(\sqrt{a}\,t^2+bt+c\sqrt{a})}{(2\sqrt{a}\,t+b)^2}dt$$

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bo'ladi.

2-misol. Ushbu

$$\int \frac{dx}{x+\sqrt{x^2+x+1}}$$

integral hisoblansin.

◄ Bu integralda

$$t = x+\sqrt{x^2+x+1}$$

almashtirishni bajaramiz. Natijada

$$x=\frac{t^2-1}{1+2t} \quad , \quad dx = 2\frac{t^2+t+1}{(1+2t)^2}dt$$

bo'lib,

$$\int \frac{dx}{x+\sqrt{x^2+x+1}} = 2\int \frac{t^2+t+1}{(1+2t)^2 t}dt$$

bo'ladi.

Agar

$$\frac{2(t^2+t+1)}{t(1+2t)^2} = \frac{2}{t}-\frac{3}{1+2t}-\frac{3}{(1+2t)^2}$$

bo`lishini e`tiborga olsak, unda

$$\int \frac{dx}{x+\sqrt{x^2+x+1}} = \int\left(\frac{2}{t}-\frac{3}{1+2t}-\frac{3}{(1+2t)^2}\right)dt = 2\ln|t| - \frac{3}{2}\ln|1+2t| + \frac{3}{2(1+2t)} + C =$$
$$= 2\ln\left|x+\sqrt{x^2+x+1}\right| - \frac{3}{2}\ln\left|1+2x+2\sqrt{x^2+x+1}\right| + \frac{3}{2(1+2x+2\sqrt{x^2+x+1})} + C$$

bo`lishi kelib chiqadi. ►

b) $c > 0$ **bo`lsin.** Bu holda (1) integralda ushbu

$$t = \frac{1}{x}(\sqrt{ax^2 + bx + c} - \sqrt{c})$$

yoki

$$t = \frac{1}{x}\left(\sqrt{ax^2 + bx + c} + \sqrt{c}\right)$$

almashtirishini bajaramiz. Unda

$$x = \frac{2\sqrt{c}\quad t - b}{a - t^2}, \qquad dx = \frac{\sqrt{c}t^2 - bt + \sqrt{c}\quad a}{(a+t)^2}dt, \qquad \sqrt{ax^2 + bx + c} = \frac{\sqrt{c}\quad t^2 - bt + a\sqrt{c}}{a - t^2}$$

bo`lib, (1) integral ratsional funksiyaning integraliga keladi:

$$\int R(x, \sqrt{ax^2 + bx + c})dx =$$
$$= \int R\left(\frac{2\sqrt{c}\,t - b}{a - t^2}, \frac{\sqrt{c}\,t^2 - bt + a\sqrt{c}}{a - t^2}\right)\left(\frac{\sqrt{c}t^2 - bt + \sqrt{c}\,a}{(a+t)^2}\right)dt$$

v) $ax^2 + bx + c$ **kvadrat uchhad turli** x_1 **va** x_2 **haqiqiy ildizga ega bo`lsin:**

$$ax^2 + bx + c = a(x - x_1)\cdot(x - x_2).$$

Bu holda (1) integralda ushbu

$$t = \frac{1}{x - x_1}\sqrt{ax^2 + bx + c}$$

almashtirishni bajaramiz. Natijada

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$$x=\frac{-ax_2+x_1t^2}{t^2-a}\quad,\qquad \sqrt{ax^2+bx+c}=\frac{a(x_1-x_2)}{t^2-a}t$$

$$dx=\frac{2a(x_1-x_2)t}{(t^2-a)^2}dt$$

bo`lib,

$$\int R(x,\sqrt{ax^2+bx+c})dx=$$

$$=\int R\left(\frac{-ax_2+x_1t^2}{t^2-a}\quad,\quad\frac{a(x_1-x_2)}{t^2-a}t\right)\cdot\frac{2a(x_1-x_2)t}{(t^2-a)^2}dt$$

bo`ladi.

3-misol. Ushbu

$$I=\int\frac{x-\sqrt{x^2+3x+2}}{x+\sqrt{x^2+3x+2}}dx$$

integral hisoblansin.

◄ Ravshanki,

$$x^2+3x+2=(x+1)\cdot(x+2).$$

SHuni e`tiborga olib berilgan integralda

$$t=\frac{1}{x+1}\sqrt{x^2+3x+2}$$

almashtirishni bajaramiz. U holda

$$x=\frac{2-t^2}{t^2-1}\quad,\qquad dx=-\frac{2tdt}{(t^2-1)^2}$$

bo`lib,

$$\int\frac{x-\sqrt{x^2+3x+2}}{x+\sqrt{x^2+3x+2}}dx=\int\frac{-2t^2-4t}{(t-2)\cdot(t-1)\cdot(t+1)^3}dt$$

bo`ladi.

Endi

$$\frac{-2t^2-4t}{(t-2)\cdot(t-1)\cdot(t+1)^3}=\frac{\frac{3}{4}}{t-1}-\frac{\frac{16}{27}}{t-2}-\frac{\frac{17}{108}}{t+1}+\frac{\frac{5}{18}}{(t+1)^2}+\frac{\frac{1}{3}}{(t+1)^3}$$

bo`lishini e`tiborga olib topamiz:

$$I=\int\frac{-2t^2-4t}{(t-2)\cdot(t-1)\cdot(t+1)^3}dt=\frac{3}{4}\int\frac{dt}{t-1}-\frac{16}{27}\int\frac{dt}{t-2}-$$

$$-\frac{17}{108}\int\frac{dt}{t+1}+\frac{5}{18}\int\frac{dt}{(t+1)^2}+\frac{1}{3}\int\frac{dt}{(t+1)^3}=\frac{3}{4}\ln|t-1|-$$

$$-\frac{16}{27}\ln|t-2|-\frac{17}{108}\ln|t+1|-\frac{5}{18}\cdot\frac{1}{t+1}-\frac{1}{6}\cdot\frac{1}{(t+1)^2}+C. \blacktriangleright$$

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