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DEFINITE INTEGRAL

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Abstract: This article provides detailed information about the definite integral, the main properties of the definite integral,the calculation of the definite integral, the Newton-Leibnis formula, the calculation of the surfaces of flat figures using the definite integral, and the applications of the definite integral in economics.
Key words: Definite integral, Newton-Leibnis formulas, calculate the arc length of the curve, labor productivity, product production volume.

The definite integral is one of the most basic operations of mathematical analysis. Areas, arc lengths, volumes, work done by a variable force, and several matters of economics are brought into definite integral.

Definition of definite integral and its geometric meaning. Let the section be a continuous function. We divide the section [a,b] into $\Delta x_i = x_i - x_{i-1}, i = 1, \text{n}$ partial sections, we select one point $c_1, c_2 \ldots, c_n$ in each partial section. At these points, we calculate the values of the $f(c_i)$ function and form the sum $f(c_1)\Delta x_1 + f(c_2)\Delta x_2 + \cdots + f(c_n)\Delta x_n$ this sum is called the integral sum for the function $y = f(x)$ on the section $[a, b]$. $\max\limits_{1\le i\le n} \Delta\, x_i{=}\lambda$

Description. If the integral sum $\sum_{i=1}^{n} f(c_i)\Delta x_i$ has a finite limit in [a,b] that does not depend on the method of dividing the section $[x_{i-1}, x_i](i = 1,2,3, \ldots, n)$ into sections $c_1, c_2, \ldots, c_n$ and the selection of points $\lambda \to 0$ in them, then this limit is called the definite integral of the function $f(x)$ on the section [a,b] and $\int_a^b f(x)dx$ is marked with a symbol. According to the definition, if the function

$\int_a^b f(x) = \lim\limits_{\lambda\to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i$ is continuous on the section $y = f(x)$ if it is [a,b] then it is integrable, that is, such a function has an exact integral.

The main properties of the definite integral.1. The definite integral of the algebraic sum of a finite number of integrable functions is equal to the algebraic sum of the definite integrals of the addends,

$\int_a^b [f_1(\mathrm{x})+f_2(\mathrm{x})-f_3(x)]dx = \int_a^b f_1\,(x)dx + \int_a^b f_2\,(x)dx - \int_a^b f_2\,(x)dx$

2.The constant multiplier can be deduced from the definite integral sign,

$$\int_a^b kf(x)dx = k\int_a^b f(x)dx$$

3.If the [a,b] section has $f(x) \geq 0,\ \int_a^b f(x)dx \geq 0$

4.If the inequality [a,b] is fulfilled in the section $f(x) \leq g(x)$,

$$\int_a^b f(x)dx \leq \int_a^b g(x)dx$$

5. c [a,b] is a point on the cross section

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

6. If the numbers m and M are the smallest and largest values of the function $y = f(x)$ on the section [a,b] respectively,

$$\mathrm{m(b\text{-}a)} \leq \int_a^b f(x)dx \leq M(b-a)$$

7.$\int_a^b f(x) = -\int_a^b f(x)dx$

8.$\int_a^a f(x)dx = 0$

9.$\int_a^b f(x)dx = \int_a^b f(t)dt = \int_a^b f(n)dn$

10. If $y = f(x)$ is continuous on the section [a,b] then a point c is found on this section such that the inequality $\int_a^b f(x)dx = f'(c)(b-a)$ holds. This is also called the mean value theorem.

Calculating the definite integral. Newton-Leibnis formula. Based on the definition of the definite integral, that is, calculating the limit of the sum of an infinite number of infinitesimals leads to a lot of difficulty. Therefore, a method based on specific binding is used.

If $F(x)$, [a,b] is one of the initial functions of the continuous function $f(x)$ on the section, the $\int_a^b f(x)dx = F(x)\ \Big|_a^b = F(b) - F(a)$

formula is appropriate, and it is called the Newton-Leibnis formula. Using this, the magnitude of the definite integral is calculated.

Thus, in order to calculate the definite integral, as in the case of the indefinite integral, it is necessary to find the initial function.

Example 1.

$\int_1^4 x^2dx$ calculate the integral.

Solving; $\int_1^4 x^2\,dx = \frac{x^3}{3}\Big|_1^4 = \frac{4^3}{3} - \frac{1}{3} = \frac{64}{3} - \frac{1}{3} = \frac{63}{3} = 21$

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Reminder; We have obtained the initial function $\frac{x^3}{3}$ of the function $y = x^2$, but the result is the same if we take an arbitrary initial function $\frac{x^3}{3} + c$ instead. It will indeed be

$\left(\frac{x^3}{3} + C\right)\Big|_1^4 = \frac{4^3}{3} + C - \left(\frac{1^3}{3} + C\right) = \frac{64}{3} + C - \frac{1}{3} - C = \frac{63}{3} = 21.$ Therefore, from now on, we get the initial function $C = 0$.

Example 2.

$\int_0^5 x\sqrt{x+4}\ dx$ calculate the integral.

Solving; We get $\sqrt{x+4} = t$ permutation, $x = t^2 - 4$, $dx = 2tdt$, and when $x = 0$, $\sqrt{0+4} = t$, $t = 2$, $\sqrt{5+4} = t$, $t = 3$. And so,

$\int_0^5 x\sqrt{x+4}\,dx = \int_2^3 (t2 - 4)t2dt = \int_2^3 (2t^4 - 8t^2)$ dt=$2\int_2^3 t^4 dt - 8\int_2^3 t^2\, dt = 2\frac{t^5}{5}\Big|_2^3 - 8\frac{t^3}{3}\Big|_2^3 = \frac{2}{3}(3^5 - 2^3) - \frac{8}{3}(3^3 - 2^3) = \frac{2}{5} * 211 - \frac{8}{3} * 19 = \frac{506}{15} = 33\frac{11}{15}$.

So, when a variable is changed in a definite integral, if its limits of integration are also changed over the variables, it should not return to the previous variable as in the indefinite integral.

Calculation of the surfaces of flat figures using the definite integral. The figure bounded by the graph of the function$y = f(x)$, $x = a$ $x = b$ by two straight lines and the axis OX is called a curved trapezoid. The face of such a curved trapezoid is calculated by the formula S=$\int_a^b ydx = \int_a^b f(x)dx$.

In general, the surface bounded by the lines $y_1 = f_1(x)$, $y_2 = f_2(x)$, $f_2(x) \geq f_1(x)$ is equal to the definite integral S_1=$\int_{x1}^{x2}[f_2\,(x) - f_1\,(\mathrm{x})]\mathrm{dx}$.

The surface bounded by the lines $x = \varphi(y), y = c, y = d, x = 0$ is calculated by the definite integral S_2=$\int_c^d xdy$=$\int_c^d \varphi(y)dy$.

Calculate the arc length of the curve. If $y = f(x)$ [a,b] is smooth in cross section in rectangular coordinate system [$y = f(x)$ derivative is continuous], the arc length of this curve is calculated using the formula $l = \int_a^b \sqrt{1 + (y')^2 dx}$.

If the curve is given by the parametric equation $\begin{cases} x = x(t) \\ y = y(t) \end{cases}$ the arc length $l = \int_a^b \sqrt{(y')^2 + (x')^2}\ dt$ is calculated by the definite integral.

If the smooth curve is given by the equation $r = r(\varphi)$ $(\alpha \leq \varphi \leq \beta)$ in polar coordinates, then the arc length is calculated by the formula

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$l = \int_{\alpha}^{\beta} \sqrt{r^2 + (r')^2}\, d\varphi.$

Calculation of the volume of the rotating body. The volume of the body formed by the rotation of the figure bounded by the lines $y = f(x),\ x = a,\ x = b,$ $y = 0$ around the OX axis is calculated by the exact integral V_x=$\pi \int_a^b y^2$dx=$\pi \int_a^b f^2$(x)dx.

The volume of the body formed by the rotation of the figure bounded by $x = \varphi(y), y = c, y = d, x = 0$ lines around the OY axis is calculated by the formula V_y=$\pi \int_c^d x^2$dy=$\pi \int_c^d \varphi^2\,(y)dy$.

Applications of the definite integral to economics. It is known that labor productivity is a variable amount during the working day. Let the labor productivity be represented by the function $y = f(x)$ where x is the time interval calculated from the beginning of the working day, and $f(x)$ is the labor productivity at this moment in time. Let the issue of labor productivity in the 4th hour of the working day be put. If we divide the time interval (3.4) into intervals, the largest of which is use $\underline{\Delta} x$ and if we say that the function $f(x)$ does not change in these small intervals, the production is equal to multiplying labor productivity by $f(x)\underline{\Delta} x$. Thus, production labor productivity in the 4th hour of the working day is represented by the equation $\lim\limits_{\Delta x \to 0} \sum_3^4 f(x)\underline{\Delta} x = \int_3^4 f(x)dx.$

If $f(x)$ is the amount of products delivered to the warehouse per unit of time, and x is the unit of time starting from the arrival of the products into the warehouse, then $x + \underline{\Delta} x$ units of products arrive at the warehouse in the time interval from x to $f(x)\underline{\Delta} x$. So, if the goods arrive in the warehouse continuously, the stock of goods in it is represented by $\int_0^x f(x)dx$.

The machine-building industry manufactures some type of machine tools, and its annual output is constant α, and let x be the number of years in which these machines are produced. The number of machines at time t [assumed to be idle] is $\int_0^t adx = [ax]_0^t$. If the volume of product production is $f(x) = a_0 + a_1 x$ which grows according to arithmetic progression, the number of machines is $\int_0^t (a_0 + a_1 x)$dx=$\left[a_0 + \frac{a_1 x^2}{2}\right]_0^t = a_0$+$\frac{a_1 t^2}{2}$

Let $D = f(x)$ be the annual income function of time t. The percentage of the rate of interest is i, and it is calculated continuously by adding on the percentages. Find the discounted volume of income calculated for t years. The discount is the difference between the final amount and the initial amount. To calculate this quantity, we divide the time interval t into equal parts. In a very small time interval

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$\triangle t$, the gain can be assumed to be constant equal to $f(t)\ \triangle t$. The discounted income in continuous compound interest is calculated as follows; $\frac{f(t)\triangle t}{e^{it}}$=$f(t)\triangle te^{-it}$. The amount of discounted income in the time interval $(0,t)$ is $\lim\limits_{\triangle t\to 0}\sum_0^t f(t)\,e^{-it}\ \triangle t =$ $\int_0^t f(t)e^{-it}\,dt$.

In particular, if the annual return is constant and $f(x)=a$, the discounted return is d=$\int_0^t ae^{-it}\,dt = a\int_0^t fe^{-it}dt = a\left[-\frac{1}{i}e^{-it}\right]_0^t = \frac{a}{i}(1-e^{-it})$.

The definite integral finds numerous applications in real-world scenarios. Here are a few examples:

1. Calculating Area: The definite integral can be used to find the area under a curve. For instance, in engineering and physics, it is employed to calculate the area of irregular shapes, such as determining the surface area of complex objects or finding the area enclosed by a given curve.

2. Physics and Kinematics: The definite integral plays a crucial role in physics, particularly in kinematics. It is used to determine displacement, velocity, and acceleration of objects by integrating the corresponding functions over time.

3. Economics and Finance: In economics, the definite integral is utilized to analyze economic indicators such as consumer surplus and producer surplus. It is also employed in finance to calculate measures like net present value, which involves integrating the cash flows over time.

4. Probability and Statistics: The definite integral is employed in probability theory and statistics to compute probabilities and cumulative distribution functions. For example, to find the probability that a random variable falls within a specific range, the corresponding probability density function is integrated over that range.

5. Engineering and Signal Processing: Engineers often use the definite integral to analyze signals and systems. It helps in determining energy, power, and other characteristics of signals by integrating their squared values over time.

6. Fluid Mechanics: In fluid mechanics, the definite integral is used to calculate quantities such as flow rate, total mass, and pressure distribution. Integration is employed to sum up infinitesimally small contributions over a given region.

To conclude, the definite integral provides a powerful tool for finding the total change, area, or accumulated quantity of a function within a given interval. By evaluating the definite integral, we can obtain precise numerical values that help us understand and analyze a wide range of phenomena. Its significance extends beyond

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pure mathematics, enabling us to solve real-world problems involving rates, areas, volumes, and more. The definite integral is a vital concept that forms the basis for further exploration in calculus and serves as a cornerstone in many scientific and technological advancements.

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