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MONOTON FUNKSIYANI LOGARIFIMIK VA RATSIONAL TENGSIZLIKLARGA TADBIQI.

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Annotatsiya: Biz bilamizki tengsizliklarni yechishda ma'lum bir yechim algoritmi mavjud va standart shakldagi tengsizlikka keltirish mumkin. Biz bu maqolada monoton funksiyani logarifimik va ratsional tengsizliklarga tadbiqini ko'rib o'tamiz.

Kalit so'zlar: monoton funksiya, ko'rsatkichli, logarifimik, ratsional tengsizliklar.

Ta'rif. $y = f(x)$ funksiya $[a, b]$ kesmada aniqlangan bo'lsin. Agar $x_1 < x_2$ shartni qanoatlantiruvchi barcha x_1 , x_2 $\in [a, b]$ lar uchun $f(x_1) < f(x_2)$ bajarilsa, u holda $f(x)$ funksiya $[a, b]$ kesmada o'suvchi deyiladi.

Ta'rif. $y = f(x)$ funksiya $[a, b]$ kesmada aniqlangan bo'lsin. Agar $x_1 < x_2$ shartni qanoatlantiruvchi barcha x_1 , x_2 $\in [a, b]$ lar uchun $f(x_1) > f(x_2)$ bajarilsa, u holda $f(x)$ funksiya $[a, b]$ kesmada kamayuvchi deyiladi.

$y = f(x)$ funksiya faqat o'suvchi yoki faqat kamayuvchi bo'ladigan intervalga funksiyaning monotonlik intervali deyiladi.

Bizga monoton o'suvchi $f(x)$ funksiya berilgan bo'lsin va a, b lar uning aniqlanish sohasiga tegishli bo'lsin, u xolda quyidagi tasdiqlar o'rinli.

1. Tasdiq.
   $f(a) > f(b)$ tengsizlik, $a > b$ tengsizlika ekivalent. Ya'ni $f(a) - f(b) > 0$ tengsizlika, $a - b > 0$ ga ekivalent.
2. Tasdiq.
   $f(a) < f(b)$ tengsizlik, $a < b$ tengsizlika ekivalent. Ya'ni $f(a) - f(b) < 0$ tengsizlika, $a - b < 0$ ga ekivalent.

Ushbu tasdiqlardan quyidagi xulosa kelib chiqadi.

Agar $f(x)$ funksiya o'suvchi bo'lsa, u xolda $f(a) - f(b)$ ni ishorasi $a - b$ argumentlar ayirmasi ishorasi bilan ustma-ust tushadi.

Masalan. Ratsionallashtirish usuli qaysi tartibda bajariladi.

Bizga

$$\frac{f(x) - f(a)}{g(x) - g(b)} > 0 \qquad (1)$$

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buyerda $f(x), g(x)$ funksiyalar monoton o'suvchi funksiyalar. U xolda funksiyalar ayirmasini argumentlar ayirmasi bilan almashtirish mumkin

$$\frac{x-a}{x-b} > 0 \qquad (2)$$

(2) tengsizlika (1) tengsizlikning natijasi deyiladi. Bu degani (2) tengsizlik (1) ning barcha yechimlarni o'z ichiga olgan holda o'ziga hos yechimlarga ham ega bo'lishi mumkin.

Bu (2) yechimlar ichida (1) yechimlarni olish uchun (2) ning yechimlari va $f(x), g(x)$ funksiyalarning aniqlanish sohasi kesishmasi qaraladi.

Misol-1. U shbu

$$\log_{x^2}(x+2) < 1$$

tengsizlikni yeching.

Yechish: Bu tengsizlikni yechish uchun logarifimik xossasidan foydalanamiz.

Bir hil asosga o'tamiz va (1), (2) tengsizliklarga ko'ra quyidagi ko'rinishga kelamiz:

$$\frac{\lg(x+2)}{lgx^2} < 1 \ => \ \frac{\lg(x+2)}{lgx^2} - 1 < 0$$

$$\frac{\lg(x+2) - lgx^2}{lgx^2 - lg1} < 0 \qquad (3)$$

(3) tengsizlikni aniqlanish sohasini qaraymiz.

$$\begin{cases} x^2 > 0, \quad x \neq 1 \\ x+2 > 0 \\ \dfrac{x+2-x^2}{x^2-1} < 0 \end{cases} => \begin{cases} (-\infty; 0) \cup (0; +\infty), \quad x \neq 1 \\ x > -2 \\ \dfrac{x^2-x-2}{(x-1)(x+1)} = \dfrac{(x-2)(x+1)}{(x-1)(x+1)} > 0 \end{cases}$$

$$\begin{cases} (-\infty; 0) \cup (0; +\infty), \quad x \neq 1 \\ (-2; +\infty) \\ \dfrac{(x-2)(x+1)}{(x-1)(x+1)} > 0 \end{cases}$$

Javob: $(-2; -1) \cup (-1; 0) \cup (0; 1) \cup (2; \infty)$.

Misol-2. Ushbu

$$\log_{\frac{2x+2}{5x-1}}(10x^2 + x - 2) \leq 0$$

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tengsizlikni yeching.

Yechish: Bu tengsizlikni yechish uchun logarifimik xossasidan foydalanamiz.

Bir hil asosga o'tamiz va (1), (2) tengsizliklarga ko'ra quyidagi ko'rinishga kelamiz:

$$\frac{\lg(10x^2+x-2)}{lg\frac{2x+2}{5x-1}} \le 0$$

$$\frac{\lg(10x^2+x-2)-lg1}{lg\frac{2x+2}{5x-1}-lg1} \le 0 \qquad (4)$$

(4) tengsizlikni aniqlanish sohasini qaraymiz.

$$\begin{cases} \frac{2x+2}{5x-1} > 0, \quad x \ne \frac{1}{5} \\ 10x^2+x-2 > 0 \\ \frac{10x^2+x-3}{\frac{2x+2}{5x-1}-1} \le 0 \end{cases} \quad => \quad \begin{cases} 2x+2 > 0, \quad x \ne \frac{1}{5} \\ (2x+1)(5x-2) > 0 \\ \frac{(5x+3)(2x-1)(5x-1)}{3(1-x)} \le 0, \quad x \ne 1 \end{cases}$$

$$\begin{cases} (-1;+\infty) > 0, \quad x \ne \frac{1}{5} \\ (-\infty;-\frac{1}{2}) \cup (\frac{2}{5};+\infty) > 0 \\ \frac{(5x+3)(2x-1)(5x-1)}{3(1-x)} \le 0, \quad x \ne 1 \end{cases}$$

Javob: $(-\infty;-1) \cup \left[-\frac{3}{5};\frac{1}{5}\right] \cup \left[\frac{1}{5};\frac{1}{2}\right] \cup \left[\frac{1}{2};+\infty\right)$.

Misol-3. Ushbu

$$\frac{\sqrt{1-3x}-1}{\sqrt{2+x}-1} < 1$$

tengsizlikni ratsionallashtirish usulini qo'llab yeching.

Yechish: Bu tengsizlikni yechish uchun (1), (2) tengsizliklardan foydalanamiz:

$$\frac{\sqrt{1-3x}-1}{\sqrt{2+x}-1} - 1 < 0 \quad => \quad \frac{\sqrt{1-3x}-1-\sqrt{2+x}+1}{\sqrt{2+x}-1} < 0$$

$$\frac{\sqrt{1-3x}-\sqrt{2+x}}{\sqrt{2+x}-\sqrt{1}} < 0$$

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$f(x) = \sqrt{x}$ funksiya o'suvchi ishorani saqlaydi.

$$\begin{cases} \dfrac{1-3x-2-x}{2+x-1} < 0 \\ 1-3x \geq 0 \\ 2+x \geq 0, \quad 2+\ x \neq 1 \end{cases} \quad => \quad \begin{cases} \dfrac{4x+1}{x+1} > 0 \\ x \leq \dfrac{1}{3}0 \\ x \geq 2, \quad x \neq -1 \end{cases}$$

Javob: $[-2;-1) \cup \left(-\frac{1}{4};\frac{1}{3}\right]$.

Misol-3. Ushbu

$$\frac{3-x-\sqrt{5-x^2}}{cos\dfrac{2x-7}{4} - cos\dfrac{x-5}{4}} \geq 0$$

tengsizlikni yeching.

Yechish: Bu tengsizlikni yechish uchun (1), (2) tengsizliklardan foydalanamiz. Bu tengsizlikda $y = cosx$ funksiya $[0;-\pi]$ da monoton o'suvchi funksiya.

Aniqlanish sohasi $5-x^2 \geq 0 \quad <=> \quad -\sqrt{5} \leq x \leq \sqrt{5}$ teng. Tengsizlikdagi $cos\frac{2x-7}{4}, cos\frac{x-5}{4}$ funksiyalarni baholaymiz.

1) $cos\frac{2x-7}{4}$ baholaymiz.

$$\frac{-2\sqrt{5}-7}{4} \leq \frac{2x-7}{4}$$

$$\frac{-\sqrt{20}-7}{4} > \frac{-\sqrt{25}-7}{4} = -\frac{12}{4} = -3$$

$$\frac{2x-7}{4} \leq \frac{2\sqrt{5}-7}{4} < 0$$

Demak $cos\frac{2x-7}{4} \quad \rightarrow \quad [-\pi;0]$ da o'suvchi.

2) $cos\frac{x-5}{4}$ baholaymiz.

$$\frac{-\sqrt{5}-7}{4} < \frac{x-5}{4} > -\pi$$

$$\frac{x-5}{4} \leq \frac{\sqrt{5}-5}{4} < 0$$

Demak $cos\frac{x-5}{4} \quad \rightarrow \quad [-\pi;0]$ da o'suvchi.

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3) $3-x,\quad -\sqrt{5}\le 3-x\le\sqrt{5}$ da yotadi va o'suvchi. Bunda $\sqrt{(3-x)^2}$ ko'rnishida yozish mumkin. $x\in[-\sqrt{5};\sqrt{5}]$, $3-x>0$.

$$\begin{cases}\dfrac{(3-x)^2-(5-x)^2}{\dfrac{2x-7}{4}-\dfrac{x-5}{4}}\ge 0\\ x\in[-\sqrt{5};\sqrt{5}]\end{cases}$$

$$\begin{cases}\dfrac{2x^2-6x+4}{\dfrac{x-12}{4}}\ge 0\\ x\in[-\sqrt{5};\sqrt{5}]\end{cases} \quad => \quad \begin{cases}\dfrac{x^2-3x+2}{x-2}\ge 0, \quad x\ne 2\\ x\in[-\sqrt{5};\sqrt{5}]\end{cases}$$

$$\begin{cases}\dfrac{(x-1)(x-2)}{(x-2)}\ge 0, \quad x\ne 2\\ x\in\left[-\sqrt{5};\sqrt{5}\right]\end{cases}$$

Javob: $[1;2)\cup\left(2;\sqrt{5}\right]$.

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