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TABIIY USULDA GRADUIRLANGAN FILIFORM LEYBNITS ALGEBRALARINING KVAZI-DIFFERENSIALLASHLARI

Musayev Sardor Xabibulla o'g'li

FAN VA TEXNOLOGIYALAR UNIVERSITETI

"Aniq fanlar" kafedrasi o'qituvchisi

sardormusayev1999@gmail.com

ANNOTATSIYA

Ushbu maqolada tabiiy usulda graduirlangan filiform Leybnits algebralarining ro'yxatini keltiramiz va bu algebralarning oddiy differensiallashi, kvazi-differensiallashini hisoblaymiz va ularning umumiy ko'rinishini topamiz.

Kalit so'zlar: Leybnits algebralari, tabiiy usulda graduirlangan filiform Leybnits algebralari**, d**ifferensiallash, kvazi-differensiallash.

ABSTRACT

In this article, we list naturally graded filiform Leibniz algebras and calculate simple differentiation, quasi-differentiation of these algebras, and find their general representation.

Key words: Leibniz algebras, naturally graded filiform Leibniz algebras, differentiation, quasi-differentiation.

АННОТАЦИЯ

В данной статье мы перечисляем естественно градуированные филиформные алгебры Лейбница, вычисляем простое дифференцирование, квазидифференцирование этих алгебр и находим их общее представление.

Ключевые слова: Алгебры Лейбница, естественно градуированные филиформные алгебры Лейбница, дифференцирование, квазидифференцирование.

KIRISH

Hozirgi kunda Li algebralarning umumlashmasi hisoblangan Leybnits algebralari sinfi jadal suratda o'rganilmoqda. Ta'kidlash joizki, Leybnits ayniyatini qanoatlantiruvchi algebralar birinchi bo'lib 1965-yilda A.Bloxning ishida D-algebralar nomi bilan kiritilgan edi. Lekin, D-algebralarni o‘rganishga unchalik e‘tibor berilmagan bo‘lib, faqatgina J.L. Lode va T.Pirashvilining ishlaridan keyingina Leybnits algebralari jadal suratda o‘rganila boshlandi va hozirgi kunga kelib bu algebralarga bag‘ishlangan bir qator maqolalar chop qilindi Leybnits algebralari o‘tgan asrning 90-yillarida fransuz matematigi J.L. Lode tomonidan ushbu

$$[x,[y,z]] = [[x,y],z] - [[x,z],y]$$

Leybnits ayniyati bilan xarakterlanadigan algebra sifatida fanga kiritilgan. 1998-yildan boshlab Leybnits algebrasining strukturaviy nazariyasini Sh.A. Ayupov va B.A. Omirovlar o‘rgana boshladi. Algebraning o‘lchami qancha kattalashgan sari, uni tavsiflash shuncha murakkab bo‘ladi. Nilpotent Leybnits algebralari bilan Ayupov Sh.A., Omirov B.A., Raximov I.S., Rixsiboev I.M., Xudoyberdiyev A.X. va boshqalar shug‘ullangan. Katta o‘lchamdagi nilpotent Li algebralarini ham o‘rganish murakkab bo‘lgani uchun, nilpotent algebralar bir necha sinflarga bo‘linadi. Masalan, nol filiform, filiform, kvazi filiform va boshqa sinflar.

So‘nggi yillarda noassotsiativ algebralarning differensiallashlari va differensiallashlarning umumlashmalari hisoblangan qator operatorlar keng o‘rganilmoqda. Xususan, kvazi-differensiallashlar tushunchalari operator algebralaridan tashqari Li va Leybnits algebralari uchun ham o‘rganildi. Ushbu maqolada kichik o’lchamli Leybnits algebralarining kvazi-differensiallashlari tushunchasi o‘rganiladi. Kichik o’lchamli Leybnits algebralarining kvazi-differensiallashlari va ularning xossalari aniqlanadi.

Ta’rif 1. F maydonda berilgan $(L,[-,-])$ algebraning ixtiyoriy x,y,z elementlari uchun quyidagi Leybnits ayniyati o‘rinli bo‘lsa:

$$[x,[y,z]] = [[x,y],z] - [[x,z],y],$$

u holda $(L,[-,-])$ algebra Leybnits algebrasi deb ataladi.

Ta’rif 2. Aytaylik, $d: L \to L$ chiziqli akslantirish bo‘lsin. Agar $(L,[-,-])$ Leybnits algebrasining ixtiyoriy elementlari uchun quyidagi tenglik bajarilsa:

$$d([x,y]) = [d(x),y] + [x,d(y)],$$

u holda d chiziqli akslantirish L Leybnits algebrasining differensiallashi deyiladi.

Barcha differensiallashlar to‘plamini $Der(L)$ kabi belgilaymiz.

Ta'rif 3. Agar $D \in End(L)$ akslantirish uchun, $\exists D', D'' \in End(L)$ akslantirishlar topilib, $\forall x, y \in L$ elementlar uchun quyidagi ayniyat bajarilsa,

$$[D(x), y] + [x, D'(y)] = D''([x, y])$$

u holda D akslantirishga L Leybnits algebrasining **umumlashgan differensillashi** deyiladi.

Ta'rif 4. Agar $D \in End(L)$ akslantirish uchun, $\exists D' \in End(L)$ akslantirish topilib, $\forall x, y \in L$ elementlar uchun quyidagi ayniyat bajarilsa,

$$[D(x), y] + [x, D(y)] = D'([x, y])$$

u holda D akslantirishga L Leybnits algebrasining **kvazi-differensillashi** deyiladi.

L Leybnits algebrasining barcha umumlashgan va kvazi differensiallashlari to'plami mos ravishda $GDer(L)$ va $QDer(L)$ kabi belgilanadi. Ta'kidlash joizki, ixtiyoriy differensiallash kvazi differensiallash bo'ladi. Biroq, kvazi-differensiallashlar oddiy differensiallash bo'lmasligi mumkin.

Endi algebraning sentroidi, kvazi-sentoidi va sentral differensiallashlari tushunchalarini aniqlaymiz.

Ta'rif 5. L Leybnits algebrasining $\forall x, y \in L$ elementlari uchun quyidagi,

$$[D(x), y] = [x, D(y)] = D([x, y])$$

ayniyatni bajaradigan $D \in End(L)$ akslantirishlarga L Leybnits algebrasining **sentroidi** deyiladi. Barcha sentroidlar to'plamini $C(L)$ bilan belgilanadi.

Ta'rif 6. L Leybnits algebrasining $\forall x, y \in L$ elementlari uchun quyidagi,

$$[D(x), y] = [x, D(y)]$$

ayniyatni bajaradigan $D \in End(L)$ akslantirishlarga L Leybnits algebrasining **kvazi-sentroidi** deyiladi. Barcha kvazi-sentroidlar to'plamini $QC(L)$ bilan belgilanadi.

Ta'rif 7. L Leybnits algebrasining $\forall x, y \in L$ elementlari uchun quyidagi,

$$[D(x), y] = [x, D(y)] = D([x, y]) = 0$$

ayniyatni bajaradigan $D \in End(L)$ akslantirishlarga L Leybnits algebrasining **sentral differensiallashi** deyiladi. Barcha sentral differensiallashlar to'plamini $ZDer(L)$ bilan belgilanadi. Ma'lumki,

$$ZDer(L) \subseteq Der(L) \subseteq QDer(L) \subseteq GDer(L) \subseteq End(L)$$

munosabat o‘rinli bo‘ladi. Shuningdek,

$$C(L) \subseteq QC(L) \subseteq QDer(L)$$

munosabat ham o‘rinli bo‘ladi.

NATIJALAR:

Ma‘lumki, har qanday n o‘lchamli tabiiy usulda graduirlangan filiform Leybnits algebralari quyidagi o‘zaro izomorf bo‘lmagan algebralardan biriga izomorf bo‘ladi:

$$F_n^1: \ [e_1, e_1] = e_3, \ [e_i, e_1] = e_{i+1}, \ 2 \le i \le n-1$$

$$F_n^2: \ [e_1, e_1] = e_3, \ [e_i, e_1] = e_{i+1}, \ 3 \le i \le n-1$$

Ma‘lumki, bu algebralarning oddiy differensiallashlar fazosining matritsalari quyidagicha bo‘ladi:

$$Der(F_n^1) = \begin{pmatrix} d_{1,1} & d_{1,2} & d_{1,3} & \dots & d_{1,n-1} & d_{1,n} \\ 0 & d_{1,1}+d_{1,2} & d_{1,3} & \dots & d_{1,n-1} & d_{2,n} \\ 0 & 0 & 2d_{1,1}+d_{1,2} & \dots & d_{1,n-2} & d_{1,n-1} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & (n-2)d_{1,1}+d_{1,2} & d_{1,3} \\ 0 & 0 & 0 & \dots & 0 & (n-1)d_{1,1}+d_{1,2} \end{pmatrix},$$

$$Der(F_n^2) = \begin{pmatrix} d_{1,1} & d_{1,2} & d_{1,3} & \dots & d_{1,n-1} & d_{1,n} \\ 0 & d_{2,2} & 0 & \dots & 0 & d_{2,n} \\ 0 & 0 & 2d_{1,1}+d_{1,2} & \dots & d_{1,n-2} & d_{1,n-1} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & (n-2)d_{1,1}+d_{1,2} & d_{1,3} \\ 0 & 0 & 0 & \dots & 0 & (n-1)d_{1,1}+d_{1,2} \end{pmatrix}$$

Teorema 1. $\boldsymbol{F_n}^{1}$ tabiiy usulda graduirlangan filiform Leybnits algebrasining $\boldsymbol{QDer(F_n{}^1)}$ kvazi-differensiallashlar fazosining matritsasi quyidagi ko‘rinishda bo‘ladi:

$$QDer(F_n{}^1) = \begin{pmatrix} d_{1,1} & d_{1,2} & d_{1,3} & \dots & d_{1,n-1} & d_{1,n} \\ 0 & d_{1,1}+d_{1,2} & d_{1,3} & \dots & d_{1,n-1} & d_{2,n} \\ 0 & d_{3,2} & d_{3,3} & \dots & d_{3,n-1} & d_{3,n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & d_{n-1,2} & d_{n-1,3} & \dots & d_{n-1,n-1} & d_{n-1,n} \\ 0 & 0 & 0 & \dots & 0 & d_{n,n} \end{pmatrix}$$

Endi $F_n{}^2$: $[e_1, e_1] = e_3$, $[e_i, e_1] = e_{i+1}$, $3 \le i \le n-1$ algebraning kvazi differensiallashini ko‘ramiz:

Teorema 2. $\boldsymbol{F_n}^{2}$ tabiiy usulda graduirlangan filiform Leybnits algebrasining $\boldsymbol{QDer(F_n{}^2)}$ kvazi-differensiallashlar fazosining matritsasi quyidagi ko‘rinishda bo‘ladi:

$$QDer(F_n{}^2) = \begin{pmatrix} d_{1,1} & d_{1,2} & d_{1,3} & \dots & d_{1,n-1} & d_{1,n} \\ 0 & d_{2,2} & 0 & \dots & 0 & d_{2,n} \\ 0 & d_{3,2} & d_{3,3} & \dots & d_{3,n-1} & d_{3,n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & d_{n-1,2} & d_{n-1,3} & \dots & d_{n-1,n-1} & d_{n-1,n} \\ 0 & d_{n,2} & 0 & \dots & 0 & d_{n,n} \end{pmatrix}$$

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