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MANBALI UMUMLASHGAN NOCHIZIQLI SHREDINGER TENGLAMASINI BIRINCHI INTEGRAL USULIDA YECHISH

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Annotatsiya: Ushbu maqolada nochiziqli Shredinger tenglamasining o‘zgarmas amplitudali yechimlari haqida ma’lumot berilgan.

Kalit so‘zlar: nochiziqli Shredinger tenglamasi, soliton yechim, birinchi integral usuli

Ma’lumki, chiziqli bo‘lmagan murakkab fizik hodisalar fizikadan biologiya, kimyo, mexanika va boshqalarga qadar ko‘plab sohalarda ishtirok etadigan chiziqli bo‘lmagan xususiy hosilali differensial tenglamalar bilan bog‘liq. Hodisalarning matematik modellari sifatida bu tenglamalarning yechimlarini tekshirish bu hodisalarni yaxshiroq tushunishga yordam beradi.

Xususiy hosilali differensial tenglamalarning aniq yechimlarini olishning ko‘plab samarali usullari yaratilgan va ishlab chiqilgan, masalan, Li simmetriyalari usuli [1], exp-funktsiya usuli [2, 3], sin-cos usuli [4, 5], kengaytirilgan tanh-coth usuli [6, 7], proyektiv Rikkati tenglama usuli [8, 9] va boshqalar.

Birinchi integral usul Feng tomonidan [10] da kommutativ algebraning halqa nazariyasiga asoslangan bo‘lib, dastlab Burgers-KdV tenglamasini yechishda taklif qilingan. So‘nggi paytlarda bu usul ko‘pchilik tenglamalar uchun keng qo‘llanilgan.

Mazkur ishda biz ushbu

$$iu_t + au_{xx} + bu|u|^2 + icu_{xxx} + id\left(u|u|^2\right)_x = ke^{i[X(\xi)-\omega t]}, \qquad (1)$$

manbali umumlashgan nochiziqli Shredinger tenglamasini yechimini topishning birinchi integral usulini ko‘rib chiqamiz.

U quyidagi qadamlarda amalga oshiriladi:

1-qadam. $u = u(x,t)$ funksiya (1) tenglamaning yechimi bo‘lsin. Quyidagi

$$u(x,t) = f(\xi), \quad \xi = x - ct, \qquad (2)$$

almashtirishni kiritamiz. Bu almashtirish bizga quyidagi o‘zgarishlardan foydalanish imkonini beradi:

$$\frac{\partial}{\partial x}(.) = \frac{\partial}{\partial \xi}(.), \quad \frac{\partial}{\partial t}(.) = -c\frac{\partial}{\partial \xi}(.), \quad \frac{\partial^2}{\partial x^2}(.) = \frac{\partial^2}{\partial \xi^2}(.), \quad \frac{\partial^2}{\partial t \partial x}(.) = -c\frac{\partial^2}{\partial \xi^2}(.), \qquad (3)$$

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Bu almashtirishlardan foydalanib (1) nochiziqli hususiy hosilali differensial tenlamani ushbu

$$H\left(u,\frac{\partial u}{\partial \xi},\frac{\partial^2 u}{\partial \xi^2},...\right)=0. \qquad (4)$$

nochiziqli oddiy diferensial tenglamaga keltiramiz. Bu yerda $u=u(\xi)$ noma'lum funksiya, H esa u o'zgruvchili va uning hosilalaridagi ko'phad.

2-qadam. Deylik (4) ning oddiy differensial tenglama yechimini quyidagicha yozish mumkin bo'lsin:

$$u(x,t)=f(\xi). \qquad (5)$$

Budan tashqari biz yangi mustaqil o'zgaruvchini ham kiritamiz

$$X(\xi)=f(\xi), \quad Y(\xi)=\frac{\partial f(\xi)}{\partial \xi}. \qquad (6)$$

3-qadam. (5) va (6) ga ko'ra (4) tenglamani nochiziqli birinchi tartibli differensial tenglamaga quyidagicha almashtirish mumkin.

$$\begin{aligned}&\frac{\partial X(\xi)}{\partial \xi}=Y(\xi),\\ &\frac{\partial Y(\xi)}{\partial \xi}=F_1(X(\xi),Y(\xi))\end{aligned} \qquad (7)$$

Agar (7) tenglamaning integrallarini topa olsak, u holda (7) tenglamaning umumiy yechimlarini bevosita topish mumkin. Bunda $\xi=\alpha(x-\upsilon t)$ haqiqiy qiymatli funksiya va $a,b,c,d,k,\alpha,\upsilon,\omega$ larning barchasi haqiqiy.

Ushbu

$$u(x,t)=\psi(\xi)e^{i[\chi(\xi)-\omega t]}, \qquad (8)$$

ko`rinishdagi tekis to`lqin yechimlarini ko`rib chiqaylik, bu yerda $\psi(\xi)$ haqiqiy funksiya. Qulaylik uchun $\chi=\beta\xi+x_0$, bu yerda $\beta$ va $x_0$ haqiqiy o`zgarmaslar va $\xi=\alpha(x-\upsilon t)+\varsigma$ deb olamiz. Shundan keyin (8) funksiya hosilalari ustida tegishli shakl almashtirishlarni bajarib, natijaning haqiqiy va mavhum qismlarini ajratib, biz quyidagi ikkita oddiy differensial tenglamani hosil qilamiz:

$$c\alpha^3\psi'''+\left(-\alpha\upsilon+2a\beta\alpha^2-3c\alpha^3\beta^2\right)\psi'+3d\alpha\psi^2\psi'=0, \qquad (9)$$

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$$\left(a\alpha^2 - 3c\psi^3\beta\right)\psi'' + \left(\alpha\beta\upsilon + \omega - a\beta^2\alpha^2 + c\alpha^3\beta^3\right)\psi + \left(b - d\alpha\beta\right)\psi^3 - k = 0. \quad (10)$$

(10) ni ξ ga nisbatan bir marta integrallab, quyidagi

$$c\alpha^2\psi''(\xi) + \left(-\upsilon + 2a\beta\alpha - 3c\alpha^2\beta^2\right)\psi(\xi) - M = 0, \quad (11)$$

tenglikni hosil qilamiz. Bu yerda M ihtiyoriy integral o`zgarmas. Ushbu $\psi(\xi)$ funksiya (10) va (11) tengliklarni qanoatlantirgani uchun biz quyidagi cheklash shartini kiritamiz:

$$\frac{a\alpha^2 - 3c\psi^3\beta}{c\alpha^2} = \frac{\alpha\beta\upsilon + \omega - a\alpha^2\beta^2 + c\alpha^3\beta^3}{-\upsilon + 2a\alpha\beta - 3c\alpha^2\beta^2} = \frac{b - d\alpha\beta}{d} = \frac{k}{M}. \quad (12)$$

(6), (7) tengliklardan foydalanib quyidagi yechimlarni hosil qilamiz:

$$X'(\xi) = Y(\xi), \quad (13)$$

$$Y'(\xi) = \left(-\frac{d}{c\alpha^2}\right)X^3(\xi) + \left(\frac{\upsilon}{c\alpha^2} - \frac{2a\beta}{c\alpha} + 3\beta^2\right)X(\xi) + \frac{M}{c\alpha^2}. \quad (14)$$

Birinchi integral metodiga ko`ra, $X(\xi)$ va $Y(\xi)$ mos ravishda (13) va (14) ning notrivial yechimlari hamda, $P(X,Y) = \sum_{i=0}^{m} a_i(X)Y^i$ $C[X,Y]$ kompleks sohadagi qisqarmas ko`phad. Shunday deb faraz qilamizki:

$$P[X(\xi), Y(\xi)] = \sum_{i=0}^{m} a_i(X(\xi))Y(\xi)^i = 0, \quad (15)$$

bu yerda $a_i(X)$, $(i = 0,1,2,\ldots,m)$ X ning ko`phadlari va $a_m(X) \neq 0$.

Ushbu (15) tenglik (13) va (14) tengliklarning birinchi integrali deb ataladi. Bo`linish teoremasi tufayli $C[X,Y]$ kompleks sohada $h(X) + g(X)Y$ ko`phad mavjud bo`lib,

$$\frac{dP}{d\xi} = \frac{\partial P}{\partial X}\frac{dX}{d\xi} + \frac{\partial P}{\partial Y}\frac{dP}{d\xi} = [h(X) + g(X)Y]\sum_{i=0}^{m} a_i(X)Y^i. \quad (16)$$

Yuqorida biz $m = 1$ va $m = 2$ deb faraz qilib, ikkita holatni ko`rib chiqdik.

1-hol. Faraz qilaylik $m = 1$ bo`lsin. U holda (16) ning ikkala tomonidagi Y^i $(i = 2,1,0)$ koeffitsiyentlarini tenglashtirib ushbu

$$a_1'(X) = g(X)a_1(X), \quad (17)$$

$$a_0'(X) = h(X)a_1(X) + g(X)a_0(X) \quad (18)$$

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$$a_1(X)\left[\left[-\frac{d}{c\alpha^2}\right]X^3+\left[\frac{\upsilon}{c\alpha^2}-\frac{2a\beta}{c\alpha}+3\beta^2\right]X+\frac{M}{c\alpha^2}\right]=h(X)a_0(X) \qquad (19)$$

tengliklarni hosil qilamiz.

$a_i(X)$ $(i=0,1,2)$ ko‘phad bo‘lgani uchun (17) ga asosan $a_1(X)$ o‘zgarmas va $g(X)=0$ degan xulosaga kelamiz. Soddalik uchun $a_1(X)=1$ deb olamiz. $h(X)$, va $a_0(X)$ ko`phadlar darajalarini muvozanatlashtirib, biz faqat $\deg(h(X))=1$ degan xulosaga kelamiz. Faraz qilaylik, $h(X)=AX+B$ va $A\neq 0$ bo‘lsin, u holda biz $a_0(X)$ ko‘phadni

$$a_0(X)=\frac{A}{2}X^2+BX+D \qquad (20)$$

ko‘rinishda topamiz. Bunda D ihtiyoriy integral o`zgarmasi.

(19) ga $a_0(X)$, $a_1(X)$ va $h(X)$ ni qo`yib, X ning barcha darajalari oldidagi koeffitsiyentlarini nolga tenglashtirib, nochiziqli algebraik tenglamalar sistemasini hosil qilamiz va uni yechish orqali ushbu yechimlarni olamiz:

$$\begin{gathered}\upsilon=-i\sqrt{2}\sqrt{c}\sqrt{d}D\alpha+2a\alpha\beta-3c\alpha^2\beta^2,\\ M=0,\quad A=-\frac{i\sqrt{2}\sqrt{d}}{\sqrt{c\alpha}},\quad B=0,\end{gathered} \qquad (21)$$

$$\begin{gathered}\upsilon=i\sqrt{2}\sqrt{c}\sqrt{d}D\alpha+2a\alpha\beta-3c\alpha^2\beta^2,\\ M=0,\quad A=\frac{i\sqrt{2}\sqrt{d}}{\sqrt{c\alpha}},\quad B=0.\end{gathered} \qquad (22)$$

(15) da (21) va (22) shartlardan foydalanib, biz quyidagiga ega bolamiz

$$Y(\xi)=\left(\pm\frac{i\sqrt{2}\sqrt{d}}{\sqrt{c\alpha}}\right)X^2(\xi)-D. \qquad (23)$$

Biz (23) ni (13) bilan birlashtirib, (13) va (14) ning aniq yechimlarini oldik. Bundan biz (1) manbaali umumlashgan nochiziqli Shredinger tenglamasining aniq harakatlanuvchi to`lqin yechimlarini quyidagicha yozishimiz mumkin:

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$$u_1(x,t)=i(-2)^{1/4}c^{1/4}\sqrt{D}\sqrt{\alpha}\times$$
$$\times\tanh\left[(1+i)d^{1/4}\sqrt{D}\left(\alpha x-\alpha\upsilon t+\varsigma-2\sqrt{c}\alpha\xi_0\right)\times\left(2^{3/4}c^{1/4}\sqrt{\alpha}\right)^{-1}\right]\times$$
$$\left(d^{1/4}\right)^{-1}\times\exp\left[i\left(\beta\{\alpha x-\alpha\upsilon t+\varsigma\}-\omega t\right)\right],$$
$$\upsilon=-i\sqrt{2}\sqrt{c}\sqrt{d}D\alpha+2a\alpha\beta-3c\alpha^2\beta^2 \qquad (24)$$
$$u_2(x,t)=i(-2)^{1/4}c^{1/4}\sqrt{D}\sqrt{\alpha}\times$$
$$\times\tan\left[(1+i)d^{1/4}\sqrt{D}\left(\alpha x-\alpha\upsilon t+\varsigma-2\sqrt{c}\alpha\xi_0\right)\times\left(2^{3/4}c^{1/4}\sqrt{\alpha}\right)^{-1}\right]\times$$
$$\left(d^{1/4}\right)^{-1}\times\exp\left[i\left(\beta\{\alpha x-\alpha\upsilon t+\varsigma\}-\omega t\right)\right],$$
$$\upsilon=i\sqrt{2}\sqrt{c}\sqrt{d}D\alpha+2a\alpha\beta-3c\alpha^2\beta^2 \qquad (25)$$

Bu yerda ξ_0 ihtiyoriy integral o`zgarmasi.

2-hol. Faraz qilaylik $m=2$ bo`lsin. U holda (16) ning ikkala tomonidagi Y^i $(i=2,1,0)$ koeffitsiyentlarini tenglashtirib ushbu

$$a_2'(X)=g(X)a_2(X), \qquad (26)$$
$$a_1'(X)=h(X)a_2(X)+g(X)a_1(X), \qquad (27)$$
$$a_0'(X)+2a_2(X)\left[\left[-\frac{d}{c\alpha^2}\right]X^3+\left[\frac{\upsilon}{c\alpha^2}-\frac{2a\beta}{c\alpha}+3\beta^2\right]X+\frac{M}{c\alpha^2}\right]=$$
$$=h(X)a_1(X)+g(X)a_0(X) \qquad (28)$$
$$a_1(X)\left[\left[-\frac{d}{c\alpha^2}\right]X^3+\left[\frac{\upsilon}{c\alpha^2}-\frac{2a\beta}{c\alpha}+3\beta^2\right]X+\frac{M}{c\alpha^2}\right]=h(X)a_0(X). \qquad (29)$$

tengliklarni hosil qilamiz.

$a_i(X)$ $(i=0,1,2)$ ko‘phad bo‘lgani uchun (26) ga asosan $a_2(X)$ o‘zgarmas va $g(X)=0$ degan xulosaga kelamiz. Soddalik uchun $a_2(X)=1$ deb olamiz. $h(X)$, va $a_0(X)$ ko`phadlar darajalarini muvozanatlashtirib, biz faqat $\deg(h(X))=1$ degan xulosaga kelamiz. Faraz qilaylik, $h(X)=AX+B$ va $A\neq 0$ bo‘lsin, u holda biz $a_1(X)$ va $a_0(X)$ ko‘phadlarni ushbu

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$$a_1(X)=\left(\frac{A}{2}\right)X^2+BX+D \qquad (30)$$

$$a_0(X)=\left(\frac{A^2}{8}+\frac{d}{2c\alpha^2}\right)X^4+\frac{1}{2}(AB)X^3+$$
$$+\left(\frac{AD+B^2}{2}-c\alpha^2+\frac{2a\beta}{c\alpha}-3\beta^2\right)X^2+\left(BD-\frac{2M}{c\alpha^2}\right)X+F \qquad (31)$$

ko‘rinishda topamiz. Bunda A,B,D va F ihtiyoriy integral o`zgarmasi.

(29) tenglikga $a_0(X)$, $a_1(X)$ $a_2(X)$ va $h(X)$ ni qo`yib, X ning barcha darajalari oldidagi koeffitsiyentlarini nolga tenglashtirib, nochiziqli algebraik tenglamalar sistemasini hosil qilamiz va uni yechish orqali ushbu yechimlarni olamiz:

$$M=0,\quad \upsilon=\frac{1}{2}\left[-i\sqrt{2}\sqrt{c}\sqrt{d}D\alpha+4a\alpha\beta-6c\alpha^2\beta^2\right],$$
$$F=\frac{D^2}{4},\quad A=-\frac{2i\sqrt{2}\sqrt{d}}{\sqrt{c}\alpha},\quad B=0, \qquad (32)$$

$$M=0,\quad \upsilon=\frac{1}{2}\left[i\sqrt{2}\sqrt{c}\sqrt{d}D\alpha+4a\alpha\beta-6c\alpha^2\beta^2\right],$$
$$F=\frac{D^2}{4},\quad A=\frac{2i\sqrt{2}\sqrt{d}}{\sqrt{c}\alpha},\quad B=0, \qquad (33)$$

(15) da (32) va (33) shartlardan foydalanib, biz quyidagiga ega bolamiz

$$Y(\xi)=\frac{\pm i\sqrt{2}\sqrt{d}X^2(\xi)-\sqrt{c}D\alpha}{2\sqrt{c}\alpha}. \qquad (34)$$

Biz (34) ni (13) bilan birlashtirib, (13) va (14) ning aniq yechimlarini oldik. Bundan biz (1) manbaali umumlashgan nochiziqli Shredinger tenglamasining aniq harakatlanuvchi to`lqin yechimlarini quyidagicha yozishimiz mumkin:

$$u_3(x,t)=(-1)^{3/4}c^{1/4}\sqrt{D}\sqrt{\alpha}\times$$
$$\times\tanh\left[\left(\frac{1}{2}+\frac{i}{2}\right)d^{1/4}\sqrt{D}\left(\alpha x-\alpha\upsilon t+\varsigma-2\sqrt{c}\alpha\xi_0\right)\left(2^{1/4}c^{1/4}\sqrt{\alpha}\right)^{-1}\right]\times$$
$$\left(2^{1/4}d^{1/4}\right)^{-1}\times\exp\left[i\left(\beta\{\alpha x-\alpha\upsilon t+\varsigma\}-\omega t\right)\right],$$

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$$\upsilon = -\frac{1}{2} i\sqrt{2}\sqrt{c}\sqrt{d} D\alpha + 2a\alpha\beta - 3c\alpha^2\beta^2, \quad (35)$$

$$u_4(x,t) = -(-1)^{3/4} c^{1/4}\sqrt{D}\sqrt{\alpha}\times$$
$$\times\tan\left[(-1)^{1/4} d^{1/4}\sqrt{D}\left(-\alpha x + \alpha\upsilon t + \varsigma + 2\sqrt{c}\alpha\xi_0\right)\left(2^{3/4}c^{1/4}\sqrt{\alpha}\right)^{-1}\right]\times$$
$$\left(2^{1/4}d^{1/4}\right)^{-1}\times\exp\left[i\left(\beta\{\alpha x - \alpha\upsilon t + \varsigma\} - \omega t\right)\right],$$

$$\upsilon = \frac{1}{2} i\sqrt{2}\sqrt{c}\sqrt{d} D\alpha + 2a\alpha\beta - 3c\alpha^2\beta^2 \quad . \quad (36)$$

Bu yerda ξ_0 ihtiyoriy integral o`zgarmasi.

(24)-(25) va (35)-(36) tenglamalar (1) manbali umumlashgan nochiziqli Shredinger tenglamasining aniq harakatlanuvchi to`lqin yechimlaridir

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