# KO’P O‘ZGARUVCHILI FUNKSIYALARI VA GIPERGEOMETRIK CHEGARAVIY MASALALAR

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Ushbu maqolada aralash parabolik-giperbolik tipdagi tenglama uchun tip o'zgarish chizig'ida integral ko'rinishdagi ulash shartli chegaraviy masalaning bir qiymatli yechilishi isbotlangan.

**Kalit so'zlar**: Aralash tipdagi tenglama; chegaraviy masala; integral ulash sharti; integral tenglamalar usuli.

Gaussning gipergeometrik funksiyasi quyidagi qator bilan ifodalanishi mumkin[1]

$$F(a,b;,c;z)=F\begin{bmatrix} a,b; \\ x \\ c; \end{bmatrix}=\sum_{n=0}^{\infty}\frac{(a)_n(b)_n z^n}{(c)_n}\cdot\frac{z^n}{n!},\ |z|<1,(*)$$

Bu erda $a,b,c$ $z$ ga bog’liq emas. Parametrlar $a,b,c$ va $z$ o‘zgaruvchisi mumkinmurakkab qiymatlarni qabul qilib, $c\neq 0,-1,-2,...,$ va deb faraz qilinadi. $(\lambda)_v$ ifodasi Pochhammer belgisini bildiradi: $(\lambda)_0:=1,\ (\lambda)_v:=\lambda(\lambda+1)...(\lambda+v-1),\ v\in N,$ gamma funksiyasi orqali $\Gamma(z)$ shaklida yozilgan $(\lambda)_v=\frac{\Gamma(\lambda+v)}{\Gamma(\lambda)},\ \ v\in\{0\}\cup N;$ $N:=\{1,2,3,...\}$ natural sonlar to‘plami.Ko‘p darajali qator berilgan bo‘lsin

$$\sum_{|k|=0}^{\infty} A(k) x_1^{k_1} x_2^{k_2} \dots x_n^{k_n},$$

bu yerda yig'indi ko'p indeksli $k := (k_1, \dots, k_n)$ bo'yicha amalga oshiriladi.manfiy bo'lmagan butun son komponentlari $k_i \geq 0,\ i = \overline{1,n},$ ular uchunodatda, $|k| := k_1 + \dots + k_{n;}$ koeffitsient $A(k)$ va o'zgaruvchilar $x_1, \dots, x_n$ mumkinmurakkab qiymatlarni qabul qiling.

**Ta'rif (1)** Ko'p darajali qator (*) agar quyidagi $n$ ta munosabat bo'lsa, gipergeometrik qator hisoblanadi

$$f_f(k) = \frac{P_j(k)}{Q_f(k)},$$

Bu yerda $P_j$ va $Q_j$ - ko'phadli ko'phadlar, mos ravishda $p_j$ va $q_j$ darajali $k\,.Q_j$ koeffitsienti $k_j + 1;$ ga ega deb taxmin qilinadi; $P_j$ va $Q_j$ mavjud emasumumiy omillar, mumkin bo'lgan $k_j + 1\ (j = \overline{1,n}).$bundan mustasno.

Ta'rif 1.1.2. $p_1, \dots, p_n,\ q_1, \dots, q_n$ sonlarning eng kattasi deyiladi.gipergeometrik qatorning tartibi (1.1.1).

**Ta'rif (2).** Agar barcha $p_1, \dots, p_n,\ q_1, \dots, q_n$ raqamlari bir xil bo'lsa, ya'ni $p_1 = \dots = p_n = q_1 = \dots = q_n$, keyin gipergeometrik qator (1.1.1) to'liq deyiladi.Gorn[2] xususan, ikkinchisining gipergeometrik qatorlarini o'rgangan. U ba'zi qatorlarga qo'shimcha ravishda dan seriyalar orqali ifodalanganligini aniqladibitta o'zgaruvchi yoki ikkita gipergeometrik qator mahsuloti orqali,ularning har biri bitta o'zgaruvchiga bog'liq bo'lib, asosan 34 ta mavjud2-tartibdagi turli konvergent qatorlar.Ikki o'zgaruvchi bo'lsa, 14 ta to'liq seriya mavjud $p_1 = q_1 = p_2 = q_2 = 2$

$$F_1(a,b,c;d;x,y) = \sum_{m,n=0}^{\infty} \frac{(a)_{m+n}(c)_n}{(d)_{m+n} m!n!} x^m y^n,\ |x|<1,\ |y|<1, \quad (1)$$

$$F_2(a,b,c;d,e;x,y) = \sum_{m,n=0}^{\infty} \frac{(a)_{m+n}(b)_m(c)_n}{(d)_m(e)_n m!n!} x^m y^n,\ |x|+|y|<1, \quad (2)$$

$$F_3(a,b;c,d;e;x,y) = \sum_{m,n=0}^{\infty} \frac{(a)_m(b)_n(c)_m(d)_n}{(e)_{m+n} m!n!} x^m y^n,\ |x|<1,\ |y|<1, \quad (3)$$

$$F_4(a,b;c;x,y)=\sum_{m,n=0}^{\infty}\frac{(a)_{m+n}(b)_{m+n}}{(c)_m(d)_n m!n!}x^m y^n,\ \sqrt{x}+\sqrt{y}<1, \quad (4)$$

$$G_1(a,b,c;x,y)=\sum_{m,n=0}^{\infty}\frac{(a)_{m+n}(b)_{n-m}(c)_{m-n}}{m!n!}x^m y^n, \quad (5)$$

$$G_2(a,b,c,d;x,y)=\sum_{m,n=0}^{\infty}\frac{(a)_m(b)_n(c)_{n-m}(d)_{m-n}}{m!n!}x^m y^n, \quad (6)$$

$$H_1(a,b,c;d;x,y)=\sum_{m,n=0}^{\infty}\frac{(a)_{m-n}(b)_{m+n}(c)_n}{(d)_m m!n!}x^m y^n, \quad (7)$$

$$H_2(a,b,c;d;e;x,y)=\sum_{m,n=0}^{\infty}\frac{(a)_{m-n}(b)_m(c)_n(d)_n}{(e)_m m!n!}x^m y^n, \quad (8)$$

va 20 ta birlashgan qatorlar mavjud to'liq qatorlar uchun chegara shakllari va ular uchun $p_1 \le q_1 = 2,\ p_2 \le q_2 = 2$, va $p_1\ va\ p_2$ bir vaqtning o'zida ikkitaga teng bo'lolmaydi, masalan,

$$\Phi_1(a,b,c;x,y)=\sum_{m,n=0}^{\infty}\frac{(a)_{m+n}(b)_m}{(c)_{m+n} m!n!}x^m y^n,\ |x|<1, \quad (9)$$

$$\Phi_2(a,b;c;x,y)=\sum_{m,n=0}^{\infty}\frac{(a)_m(b)_m}{(c)_{m+n} m!n!}x^m y^n, \quad (10)$$

$$\Phi_3(a;d;x,y)=\sum_{m,n=0}^{\infty}\frac{(a)_m}{(d)_{m+n} m!n!}x^m y^n, \quad (11)$$

$$\Psi_1(a,b;c,d;x,y)=\sum_{m,n=0}^{\infty}\frac{(a)_m}{(c)_m(d)_n m!n!}x^m y^n,\ |x|<1, \quad (12)$$

$$\Psi_2(a;b,d;x,y)=\sum_{m,n=0}^{\infty}\frac{(a)_{m+n}}{(b)_m(d)_n m!n!}x^m y^n, \quad (13)$$

$$\Xi_1(a,b,c;d;x,y)=\sum_{m,n=0}^{\infty}\frac{(a)_m(b)_n(c)_m}{(d)_{m+n} m!n!}x^m y^n,\ |x|<1, \quad (14)$$

$$\Xi_2(a,b;d;x,y)=\sum_{m,n=0}^{\infty}\frac{(a)_m(b)_m}{(d)_{m+n} m!n!}x^m y^n,\ |x|<1, \quad (15)$$

$$\Gamma_1(a,b,c;x,y)=\sum_{m,n=0}^{\infty}\frac{(a)_m(b)_{n-m}(c)_{m-n}}{m!n!}x^m y^n,\ |x|<1, \quad (16)$$

$$\Gamma_2(b,c;x,y)=\sum_{m,n=0}^{\infty}\frac{(b)_{n-m}(c)_{m-n}}{m!n!}x^m y^n, \quad (17)$$

$$H_1(a,b;d;x,y)=\sum_{m,n=0}^{\infty}\frac{(a)_{m-n}(b)_{m+n}}{(d)_m m!n!}x^m y^n,\ |x|<1, \quad (18)$$

$$H_2(a,b,c;d;x,y)=\sum_{m,n=0}^{\infty}\frac{(a)_{m-n}(b)_m(c)_n}{(d)_m m!n!}x^m y^n,\ |x|<1, \quad (19)$$

$$H_3(a,b;d;x,y)=\sum_{m,n=0}^{\infty}\frac{(a)_{m-n}(b)_m}{(d)_m m!n!}x^m y^n,\ |x|<1, \quad (20)$$

Foydalanilgan adabiyotlar

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