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GIPERGIOMETRIK FUNKSIYALARINI VA ULARNING CHEGARAVIY MASALALARI HAQIDA

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Kalit so'zlar: Gorn ro'yxati; ikkinchi tartibli to'la va konfluent gipergeometrik funksiyalar; Srivastava-Karlsson ro'yxati; uch o'zgaruvchili konfluent gipergeometrik funksiyalar.

Ushbu maqolada ba'zi asosiy faktlar keltirilgan. Maqolaning asosiy natijalarini taqdim etish uchun zarur bo'lgan bir nechta o'zgaruvchilarning gipergeometrik funksiyalari va Burchnell-Chandy operatorlari keltirilgan.

Ushbu ikki tizim o'rtasidagi munosabat faqat ikkita o'zgaruvchili holatda yaxshi o'rganilgan. Hatto klassik Horn, Appell, Poxgammer va Lauricella, ko'p o'zgaruvchan gipergeometrik funksiyalarda ham faqat 1970 va 80-yillarda V.Miller va uning shogirdlari tomonidan differensial tenglamalarning Li algebrasini o'rganishga urinish bo'lgan. So'nggi o'ttiz yil ichida gipergeometrik funksiyalarni o'rganishga bo'lgan qiziqish ortib bormoqda. Darhaqiqat, ma'lumotlar bazasida gipergeometrik sarlavhali so'zni qidirish natijasida 3181 ta maqola topiladi, ulardan 1530 tasi 1990 yildan beri nashr etilgan ushbu yangi qiziqish gipergeometrik funksiyalar va matematikaning ko'plab sohalari o'rtasidagi bog'liqlikdan kelib chiqadi, masalan, algebraik geometriya, kombinatorika, raqamlar nazariyasi, simmetrik aks ettirishlar va boshqalar.

Ma'lumki, Gamma funksiyasi $\Gamma(s)$ quyidagi integral bilan aniqlanadi:

$$\Gamma(s)=\int_0^\infty e^{-t}t^{x-1}dt \qquad (1)$$

(1) integral $\operatorname{Re}(s)>0$ yarim tekislikda golomorf funksiyani ifodalaydi va bundan tashqari u quyidagi funksional tenglamani qanoatlantiradi.

$$\Gamma(s+1)=s\Gamma(s);\quad \operatorname{Re}(s)>0 \qquad (2)$$

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Demak, $\Gamma(1)=1$ bo'lgani uchun $\Gamma(n+1)=n!,\ (n\in N)$ ekanligi kelib chiqadi.

$\Gamma(s)$ ni musbat bo'lmagan butun sonlarda oddiy qutblar bilan butun kompleks tekislikdagi meromorf funksiyaga kengaytirish uchun (2) dan foydalanishimiz mumkin.

Masalan, $\{-1<\mathrm{Re}(s)\le 0\}$ palasada $\Gamma(s)$ ni quyidagicha aniqlaymiz:

$$\Gamma(s)=\frac{\Gamma(s+1);}{s}$$

Ta'rif. $\alpha\in C/Z_{\le 0}$ va $k\in N$ ni hisobga olgan holda biz Poxgammer belgisini aniqlaymiz:

$$(\alpha)_k=\frac{\Gamma(a+k);}{\Gamma(a)} \qquad (3)$$

Aytaylik $n=(n_1,n_2,......n_r)\in N^r$ manfiy bo'lmagan butun sonlarning *r*-tartiblisi

bo'lsin. Xuddi shu kabi $x=(x_1,x_2,......x_r)\in C^r$ berilgan bo'lsa, biz buni

x^n bilan belgilaymiz.

U holda

$x^n=(x_1^{n_1},x^{n_2}{}_2,......x^{n_r}{}_r)$ va Q^r dagi *j*- tartibli standart bazis vektorini e_j bilan belgilaymiz.

Ta'rif . Barcha $j=1,2,........,r$ lar uchun,

$R_j(n)=\dfrac{A_{n+e_j}}{A_n}$ nisbat $n=(n_1,n_2,......n_r)$ ning ratsional funksiyasi bo'lsa, ushbu ko'p o'zgaruvchili darajali qator

$$F(x_1,x_2,.....,x_r)=\sum_{n\in N^r}^{\infty}A_n x^n$$

Gorn gipergeometrik funksiyasi deyiladi.

Quyidagi qatorlar odatda Gauss gipergeometrik qatori deb ataladi

$${}_2F_1(\alpha,\beta,\gamma;x)=\sum_{n=0}^{\infty}\frac{(\alpha)_n(\beta)_n}{(\gamma)_n\,n!}x^n;\gamma\in Z_{\le 0} \qquad (4)$$

Gipergeometrik qator koeffitsiyentlarining rekurrent xossalari ularning oddiy yoki xususiy hosilali differensial tenglamalarning yechimini ifodalashga imkon beradi.

Gaussning gipergeometrik funksiyasi qanoatlantiradigan ikkinchi tartibli oddiydifferensial tenglamani keltirib chiqarishni koradigan bo'lsak, bunda biz quyidagio'zgartirishlardan foydalanamiz: bir x o'zgaruvchining funksiyasi uchun biz $\frac{d}{dx}$ differentsiallash operatorini ∂_x orqali, bir necha $(x_1, x_2, \ldots\ldots x_r)$ o'zgaruvchilarning funksiyalari uchun biz $\frac{\partial}{\partial x_j}$ xususiy hosila operatori uchun

∂_j dan foydalanamiz. Yoki soddalik uchun quyidagi Eyler operatorlaridan foydalanishimiz mumkin:

$$\theta_x = x\partial_x; \theta_j = x_j\partial_j$$

Endi Gaussning gipergeometrik (4) qatorini ko'rib chiqaylik. Ma'lumki,

$$\theta_x F(\alpha,\beta,\gamma;x) = \sum_{m,n=0}^{\infty} \frac{(\alpha)_n(\beta)_n}{(\gamma)_n n!} x^n;$$

Lekin, $n(\alpha)_n = \alpha((\alpha+1) - (\alpha)_n)$ ga muvofiq

$$\theta_x F(\alpha,\beta,\gamma;x) = \alpha \sum_{m,n=0}^{\infty} \left(\frac{(\alpha+1)_n(\beta)_n}{(\gamma)_n n!} - \frac{(\alpha)_n(\beta)_n}{(\gamma)_n n!} \right) x^n = \alpha\left(F(\alpha+1,\beta,\gamma;x) - F(\alpha,\beta,\gamma;x)\right)$$

.

Demak,

$$(\theta_x + \alpha) F(\alpha,\beta,\gamma;x) = \alpha F(\alpha+1,\beta,\gamma;x)$$

$$(\theta_x + \beta) F(\alpha,\beta,\gamma;x) = \alpha F(\alpha,\beta+1,\gamma;x)$$

Xuddi shu kabi, quyidagi tenglikka ham ega bo'lamiz:

$$(\theta_x + (\gamma-1)) F(\alpha,\beta,\gamma;x) = (\gamma-1) F(\alpha,\beta,\gamma-1;x)$$

$$\partial_x F(\alpha,\beta,\gamma;x) = \frac{\alpha\beta}{\gamma} F(\alpha+1,\beta+1,\gamma+1;x)$$

Yuqoridagi to'rtta tenglamani birlashtirib, Gaussning gipergeometrik qatori quyidagi oddiy differensial tenglamani qanoatlantirishini topishimiz mumkin bo'ladi:

$$\left(\theta_x+\alpha\right)\left(\theta_x+\beta\right)F=(\theta_x+\gamma)\partial_x F \tag{5}$$

(5) tenglama ushbu tenglamaga ekvivalent ekanligini ko'rishimiz mumkin:

$$x(x-1)\partial_x^2 F+((\alpha+\beta+1)x-\gamma)\partial_x F+\alpha\beta F=0$$

Shunday usulda gipergeometrik funksiyalar va ma'lum formulalar orqali bir nechta o'zgaruvchilarning funksiyalari ko'rinishida yoyib chiqish sirlari o'rganib chiqilgan va haligacha o'rganilmoqda.

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