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Kasr tartibli differensial operator ishtirok etgan integro-differensial tenglamalar uchun integral shartli masalalar

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Annotatsiya

Ushbu maqolada kasr tartibli differensial operator qatnashgan integro-differensial tenglama uchun ikkinchi tur integral shartli masala o‘rganilgan.

1-masala.

$$y''(x)+p_1(x)y'(x)+p_2(x)y(x)+p_3(x)D_{ax}^{\alpha}\omega(x)y(x)=f(x),$$
$$x\in(a,b)$$
(1)

tenglamaning $[a,b]$ *segmentda aniqlangan, uzluksiz va*

$$y(a)=k_1, \qquad y'(b)+hy(b)=h\int_{\alpha}^{\beta}y(t)dt+k_2 \tag{2}$$

shartlarni qanoatlantiruvchi yechimi topilsin, bu yerda k_1,k_2,h,α,β-berilgan sonlar bo‘lib, $a\le\alpha<\beta\le b$.

(2) dan ko‘rinib turibdiki $h=0$ da 1-masaladan

$$y(a)=k_1 \qquad y'(b)=k_2 \tag{3}$$

chegaraviy shartlarni qanoatlantiruvchi masala kelib chiqadi. Agar $0<|h|\le 1$ va $a<\alpha<\beta<b$ bo‘lsa, (2) shartlarning ikkinchisini undagi integralga o‘rta qiymat haqidagi teoremani tatbiq qilib, $y'(b)+hy(\xi)=k_2$ ko‘rinishda yozib olish mumkin bo‘ladi, bu yerda ξ - $[a,b]$ segmentdagi qandaydir tayinlangan son. Demak, bu holda, 1-masala

$$y(a)=k_1, \qquad y'(b)+hy(\xi)=k_2 \tag{4}$$

shartlarni qanoatlantiruvchi yechimi topilganidek o‘rganiladi. $0<|q|<1$ va $[\alpha,\beta]=[a,b]$ bo‘lgan holda ham 1-masala 4-shartga keltirib, o‘rganiladi.

Yuqoridagilarni e'tiborga olgan holda 1-masalani $h=1$, $\alpha=a$, $\beta=b$ bo'lgan holda, ya'ni (2) shartlar

$$y(a)=k_1 \qquad y'(b)+y(b)=\int_a^b y(t)dt+k_2 \tag{5}$$

ko'rinishga ega bo'lgan holda o'rganamiz.

Bu masalaning yechimi mavjud va yagonaligini ko'rsatish uchun xuddi 3-shartdagi kabi, (1) tenglamani $[a,x]$ oraliqda ikki marta integrallab,

$$y'(x)+p_1(x)y(x)+\int_a^x\left\{p_2(t)-p_1'(t)+\frac{\omega(t)}{\Gamma(1-\alpha)}\left[p_3(x)(x-t)^{-\alpha}-\right.\right.$$
$$\left.\left.-\int_t^x p_3'(z)(z-t)^{-\alpha}dz\right]\right\}y(t)dt=\int_a^x f(t)dt+y'(a)+k_1p_1(a), \tag{6}$$

va

$$y(x)+\int_a^x\{p_1(t)+[p_2(t)-p_1'(t)](x-t)+$$
$$\left.+\frac{\omega(t)}{\Gamma(1-\alpha)}\int_t^x(\xi-t)^{-\alpha}\left[p_3(\xi)-p'_3(\xi)(x-\xi)d\xi\right]y(t)dt\right\}= \tag{7}$$
$$=\int_a^x(x-t)f(t)dt+y'(a)(x-a)+k_1p_1(a)(x-a)+k_1$$

tengliklarga ega bo'lamiz. (6) va (7) tengliklarni quyidagicha yozib olamiz;

$$y(x)=-\int_a^x\Big\{p_1(t)+[p_2(t)-p_1'(t)](x-t)+$$

$$+\frac{\omega(t)}{\Gamma(1-\alpha)}\int_t^x(\xi-t)^{-\alpha}\left[p_3(\xi)-p'_3(\xi)(x-\xi)d\xi\right]y(t)dt\Big\}+$$

$$+\int_a^x(x-t)f(t)dt+y'(a)(x-a)+k_1p_1(a)(x-a)+k_1,$$

$$y'(x)=-p_1(x)y(x)-\int_a^x\Big\{p_2(t)-p_1'(t)+\frac{\omega(t)}{\Gamma(1-\alpha)}\Big[p_3(x)(x-t)^{-\alpha}-$$

$$-\int_t^x p_3'(z)(z-t)^{-\alpha}dz\Big]\Big\}y(t)dt+\int_a^x f(t)dt+y'(a)+k_1p_1(a).$$

Bulardan quyidagiga ega bo'lamiz:

$$y(x)=\int_a^x K_2(x,t)y(t)dt+f_2(x)+y'(a)(x-a),$$

(8)

$$y'(x)=-p_1(x)y(x)-\int_a^x K_1(x,t)y(t)dt+f_1(x)+y'(a),$$

(9)

bu yerda

$$f_1(x)=\int_a^x f(t)dt+k_1p_1(a),\qquad f_2(x)=\int_a^x(x-t)f(t)dt+k_1p_1(a)(x-a)+k_1,$$

$$K_1(x,t)=p_1(t)+[p_2(t)-p_1'(t)](x-t)+$$

$$+\frac{\omega(t)}{\Gamma(1-\alpha)}\int_t^x(\xi-t)^{-\alpha}\left[p_3(\xi)-p'_3(\xi)(x-\xi)d\xi\right],$$

$$K_2(x,t)=-p_1(t)+\left[p_2(t)-p_1'(t)\right](x-t)-$$

$$-\frac{\omega(t)}{\Gamma(1-\alpha)}\int(\xi-t)^{-\alpha}\left[p_3(\xi)-p'_3(\xi)(x-\xi)\right].$$

$K_1(x,t)$, $K_2(x,t)-\{(x,t): a \le x \le b\}$ to'rtburchakda chegaralangan va bo'lakli uzluksiz bo'lgan ma'lum funksiyalar, $f_1(x)$, $f_2(x)$ esa $[a,b]$ da uzluksiz ma'lum funksiyalar.

(8) dan quyidagini topamiz:

$$\int_a^b y(t)\,dt = \int_a^b\int_a^b K_2(x,t)y(t)dt + \int_a^b f_2(t)dt + \frac{1}{2}y'(a)(b-a)^2.$$

Buni va

(8) va (9) tengliklarda $x=b$ deb,

$$y'(b) = -p_1(b)y(b) - \int_a^b K_1(b,t)y(t)dt + f_1(b) + y'(a),$$

(10)

$$y(b) = \int_a^b K_2(b,t)y(t)dt + f_2(b) + y'(a)(b-a),$$

(11)

tengliklarni topamiz. (10) va (11) ni (5) shartga qo'yib,

$$y'(a)\cdot\{(b-a)[-p_1(b)+1]+1\} =$$
$$= -[-p_1(b)+1]\cdot\int_a^b K_2(b,t)y(t)dt + \int_a^b K_1(b,t)y(t)dt -$$
$$-f_2(b)[-p_1(b)+1] - f_1(b) + \int_a^b y(t)dt + k_2$$

tenglikka ega bo'lamiz.

Agar $(b-a)[-p_1(b)+1]+1 \ne 0$ bo'lsa $y'(a)$ oxirgi tenglikdan bir qiymatli topiladi

$$y'(a) = \left\{-[-p_1(b)+1]\cdot\int_a^b K_2(b,t)y(t)dt + \int_a^b K_1(b,t)y(t)dt - \right.$$
$$\left. -f_2(b)[-p_1(b)+1] - f_1(b) + \int_a^b y(t)dt + k_2\right\}\cdot\frac{1}{(b-a)[-p_1(b)+1]+1}.$$

Uni (8) ga qo'yib,

$$y(x)=\int_a^x K_2(x,t)y(t)dt+f_2(x)+\Big\{-\big[-p_1(b)+1\big]\cdot\int_a^b K_2(b,t)y(t)dt+\int_a^b K_1(b,t)y(t)dt-$$
$$-f_2(b)\big[-p_1(b)+1\big]-f_1(b)+\int_a^b y(t)dt+k_2\Big\}\cdot\frac{x-a}{(b-a)\big[-p_1(b)+1\big]+1}$$

(12) $y(x)$ ga nisbatan ikkinchi tur Fredgolm integral tenglamasiga ega bo‘lamiz.

Agar berilganlarga qo‘yilgan $f(x)\in C[a,b]$, $p_1(x)\in C^1[a,b]$, $p_2(x), p_3(x)\in C^2[a,b]$ shartlarda $|K_2(x,t)|<1$ bo‘lsa, (12) tenglama va, demak, 1-masala yagona yechimga ega bo‘ladi.

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