2-TOM, 1-SON

SOCHILISH NAZARIYASINING TESKARI MASALASI

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Annotatsiya: ushbu maqolada sochilish nazariyasining berilganlari orqali u(x) potensialni topish, shu bilan birga sochilish nazariyasining berilganlari orqali $u(x)$ potensialni topish imkonini beradigan asosiy integral tenglama keltirilgan

Kalit so'zlar: xos qiymat,Loran qatori, xos funksiya, sochilish nazariyasining berilganlari, integral tenglama,

Kirish:

Sochilish nazariyasining teskari masalasi deb, sochilish nazariyasining berilganlari orqali $u(x)$ potensialni topishga aytiladi.

$a(\xi)$ $(\operatorname{Im}\xi > 0)$ funksiyani $r(\xi) = \dfrac{b(\xi)}{a(\xi)}$ funksiya va L operatorning ξ_k ($\operatorname{Im}\xi_k > 0, \quad k = 1,2,...,N$) xos qiymatlari orqali

$$a(\xi) = \exp\left\{-\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{\ln(1+|r(\varsigma)|^2)}{\varsigma-\xi}d\varsigma\right\}\prod_{k=1}^{n}\frac{\xi-\xi_k}{\xi-\overline{\xi}_k}$$

qurish mumkin.

Bu tenglik L operator karrali xos qiymatga ega holida, ushbu

$$a(\xi) = \exp\left\{-\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{\ln(1+|r(\varsigma)|^2)}{\varsigma-\xi}d\varsigma\right\}\prod_{k=1}^{n}\left(\frac{\xi-\xi_k}{\xi-\overline{\xi}_k}\right)^{m_k}$$

ko'rinishda bo'ladi.

Endi biz sochilish nazariyasing berilganlari yordamida $u(x)$ potensialni topish imkonini beradigan asosiy integral tenglamani keltirib chiqaramiz.

Buning uchun **Ошибка! Источник ссылки не найден.** asimptotik formulalarga asosan

$$r(\xi) = \frac{b(\xi)}{a(\xi)} = \underline{\underline{O}}\left(\frac{1}{|\xi|}\right) \qquad (1)$$

asimtotika o'rinli bo'lishini topamiz. Bundan esa $r(\xi) \in L^2(-\infty,\infty)$ bo'lishi kelib chiqadi.

Shuning uchun ushbu

$$F_1(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty} r(\xi)e^{i\xi x}dx$$

Furye almashtirishi mavjud bo'lib, $F_1(x)\in L^2(-\infty,\infty)$ bo'ladi.

Quyidagi

$$S_{11}(\xi)=\frac{1}{a(\xi)} \qquad (2)$$

belgilashni kiritamiz. Ravshanki, $S_{11}(\xi)$ funksiya yuqori yarim tekislikka meromorf davom qiladi va u yerda $\xi_1,\xi_2,...,\xi_N$ cheklita mos ravishda $m_1,m_2,...,m_N$ karrali bo'lgan qutblarga ega bo'ladi. (2) funksiyani $0<|\xi-\xi_n|<\rho_k$ ($k=1,2,...,N$) xalqalarda Loran qatoriga yoyamiz:

$$S_{11}(\xi)=(\xi-\xi_k)^{-m_k}S_{m_k}^{(k)}+(\xi-\xi_k)^{-m_k+1}S_{m_k-1}^{(k)}+...+(\xi-\xi_k)^{-1}S_1^{(k)}+S_0^{(k)}+...\ ,$$
$$k=1,2,...,N$$

Xos funksiya va yopishgan funksiyalar aniqlanishiga ko'ra shunday noldan farqli $R_1^{(k)},R_2^{(k)},...,R_{m_k}^{(k)}$ sonlar mavjud bo'lib, quyidagi munosabatlar o'rinli bo'ladi:

$$\overset{0}{\varphi}(x,\xi_k)S_{m_k}^{(k)}=\overset{0}{\psi}(x,\xi_k)R_{m_k}^{(k)},$$
$$\overset{0}{\varphi}(x,\xi_k)S_{m_k-1}^{(k)}+\overset{(1)}{\varphi}(x,\xi_k)S_{m_k}^{(k)}=\overset{0}{\psi}(x,\xi_k)R_{m_k-1}^{(k)}+\overset{(1)}{\psi}(x,\xi_k)R_{m_k}^{(k)},$$
$$.\ .\ .\ .\ .\ .\ .\ . \qquad (3)$$
$$\overset{0}{\varphi}(x,\xi_k)S_1^{(k)}+\overset{(1)}{\varphi}(x,\xi_k)S_2^{(k)}+...+\frac{1}{(m_k-1)!}\overset{(m_k-1)}{\varphi}(x,\xi_k)S_{m_k}^{(k)}=$$
$$=\overset{0}{\psi}(x,\xi_k)R_1^{(k)}+\overset{(1)}{\psi}(x,\xi_k)R_2^{(k)}+...+\frac{1}{(m_k-1)!}\overset{(m_k-1)}{\psi}(x,\xi_k)R_{m_k}^{(k)},\quad k=1,2,...,N$$

$\{r^+(\xi)\equiv\frac{b(\xi)}{a(\xi)},\ \xi_k,R_1^{(k)},R_2^{(k)},...,R_{m_k}^{(k)},\ k=1,2,...,N\}$ tizimga L operatorning sochilish nazariyasining berilganlari deyiladi. Ravshanki, $R_j^{(k)}$ lar $\{\chi_1^{(k)},\chi_2^{(k)},...,\chi_{m_k}^{(k)}\}$ ($j=1,...,m_k$) normallovchi sonlar tizimi orqali chiziqli ifodalanadi. Ushbu

$$\begin{cases}\varphi=a(\xi)\overline{\psi}+b(\xi)\psi,\\ \overline{\varphi}=-\overline{a}(\xi)\psi+\overline{b}(\xi)\overline{\psi}\end{cases}$$ tengliklarning birinchi

$$\varphi=a(\xi)\overline{\psi}+b(\xi)\psi$$

tengligini quyidagi

$$\frac{1}{a(\xi)}\varphi = \overline{\psi} + r^{+}(\xi)\psi$$

ko'rinishda yozib olamiz. Oxirgi tenglikka ψ funksiya uchun $\psi(x,\xi) = \begin{pmatrix}0\\1\end{pmatrix}e^{i\xi x} + \int_x^\infty K(x,t)e^{i\xi t}dt$ Levin tasvirini qo'yib

$$S_{11}(\xi)\varphi(x,\xi) = \begin{pmatrix}0\\1\end{pmatrix}e^{-i\xi x} + \int_x^\infty \overline{K}(x,s)e^{-i\xi s}ds + \begin{pmatrix}0\\1\end{pmatrix}r^{+}(\xi)e^{i\xi x} + \int_x^\infty r^{+}(\xi)K(x,s)e^{i\xi x}ds \quad (4)$$

tenglikni xosil qilamiz. Bu tenglikning ikkala tarafini ham

$$\frac{1}{2\pi}\frac{\sin\varepsilon\xi}{\varepsilon\xi}e^{i\xi x} \qquad (z > x, \quad 0 < \varepsilon < \frac{z-x}{2})$$

funksiyaga ko'paytirib ξ bo'yicha $-\infty$ dan ∞ integrallaymiz va $\varepsilon \to 0$ da limitga o'tamiz. Unda (4) tenglikning o'ng tarafi quyidagi ko'rinishda bo'ladi:

$$RHS = \overline{K}(x,z) + F_1(x+z)\begin{pmatrix}0\\1\end{pmatrix} + \int_x^\infty K(x,s)F_1(s+z)ds. \qquad (5)$$

Bu yerda

$$F_s(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} r^{+}(\xi)e^{i\xi x}d\xi. \qquad (6)$$

Chap tarafi ***Jordan*** lemmasiga ko'ra yuqori yarim tekislikda $\varepsilon < \dfrac{z-x}{2}$ da

$$\lim_{\varepsilon\to 0}\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\sin\varepsilon\xi}{\varepsilon\xi}\varphi(x,\xi)S_{11}(\xi)e^{i\xi z}d\xi = i\sum_{k=1}^{N}\underset{\xi=\xi_k}{res}\{\varphi(x,\xi)S_{11}(\xi)e^{i\xi z}\} \qquad (7)$$

bo'ladi. $\varphi(x,\xi)e^{i\xi z}$ funksiyani ξ_k nušta atrofida Teylor qatoriga yoyamiz:

$$\varphi(x,\xi)e^{i\xi z} = \sum_{j=0}^{m_k-1}\frac{(\xi-\xi_k)^j}{j!}\left\{\sum_{l=0}^{j}C_j^l \overset{(j-l)}{\varphi}(x,\xi_k)(iz)^l\right\}e^{i\xi_n z} + \overline{\overline{o}}((\xi-\xi_k)^{m_k-1}).$$

Unda ($\underset{z=z_0}{res}\, f(z) = c_{-1}$ bo'lgani uchun)

$$R \equiv \underset{\xi=\xi_k}{res}\{\varphi(x,\xi)S_{11}(\xi)e^{i\xi z}\} = \sum_{j=0}^{m_k-1}\left\{\sum_{l=0}^{j}\frac{(iz)^l}{l!(j-l)!}\overset{(j-l)}{\varphi}(x,\xi_k)S_{j+1}^{(k)}\right\}e^{i\xi z}$$

bo'ladi. Buni (3) tengliklardan foydalanib

$$R \equiv \sum_{j=0}^{m_k-1}\left\{\sum_{l=0}^{j}\frac{(iz)^l}{l!(j-l)!}\overset{(j-l)}{\psi}(x,\xi_k)R_{j+1}^{(k)}\right\}e^{i\xi_n z} = \sum_{j=0}^{m_k-1}\frac{(iz)^l}{l!}\left\{\sum_{j=l}^{m_k-1}\frac{1}{(j-l)!}\overset{(j-l)}{\psi}(x,\xi_k)R_{j+1}^{(k)}\right\}e^{i\xi_n z}$$

(8)

ko'rinishda yozish mumkin.

Agar oxirgi tenglikda $\psi(x,\xi)=e^{i\xi x}$ desak, u holda

$$\sum_{j=0}^{m_k-1}\frac{(iz)^l}{l!}\left\{\sum_{j=l}^{m_k-1}\frac{(ix)^{j-l}}{(j-l)!}R_{j+1}^{(k)}e^{i\xi_n(z+x)}\right\}=\sum_{j=0}^{m_k-1}\frac{1}{j!}R_{j+1}\left\{\sum_{l=0}^{j}C_j^l(iz)^l(ix)^{j-l}\right\}e^{i\xi_n(z+x)}=$$

$$=\sum_{j=0}^{m_k-1}\frac{R_{j+1}}{j!}(i(z+x))^j e^{i\xi_n(z+x)}.$$

tenglikni xosil qilamiz.

Agar $R_k(x)=\sum_{j=0}^{m_k-1}\frac{(ix)^j}{j!}R_{j+1}e^{i\xi_n x}$ belgilash kiritsak, u holda

$\psi(x,\xi)=\begin{pmatrix}0\\1\end{pmatrix}e^{i\xi x}+\int_x^\infty K(x,t)e^{i\xi t}dt$ ga Levin tasvirini qo'yib

$$R=\begin{pmatrix}0\\1\end{pmatrix}R_k(x+z)+\int_x^\infty K(x,s)R_k(s+z)ds$$

tenglikni xosil qilamiz.

(4) tenglikning chap tarafini integrallagandan keyin

$$i\sum_{k=1}^{N}\left(\begin{pmatrix}0\\1\end{pmatrix}R_k(x+z)+\int_x^\infty K(x,s)R_k(s+z)ds\right)$$

ko'rinishga keladi. Ushbu $F_k(x)=-i\sum_{j=l}^{m_k-1}\frac{(ix)^{j-l}}{(j-l)!}R_{j+1}^{(k)}e^{i\xi_n x}$ va

$$F(x)=\sum_{k=1}^{N}F_k(x)+F_s(x) \qquad (9)$$

belgilashlar kiritsak quyidagi

$$\overline{K}(x,z)+\begin{pmatrix}0\\1\end{pmatrix}F(x+z)+\int_x^\infty K(x,s)F(s+z)ds=0$$

integral tenglama kelib chiqadi. Buni

$$\begin{cases}K_2(x,z)+\int_x^\infty K_1(x,s)F(s+z)ds=0\\ F(x+z)-K_1(x,z)+\int_x^\infty K_2(x,s)F(s+z)ds=0\end{cases} \qquad (10)$$

ko'rinishda ham yozish mumkin.

Xulosa:

ushbu maqolada sochilish nazariyasining berilganlari orqali u(x) potensialni topish, shu bilan birga sochilish nazariyasining berilganlari orqali $u(x)$ potensialni topish imkonini beradigan asosiy integral tenglama keltirilgan

Foydalanilgan adabiyotlar

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