GELMGOLTS TENGLAMASI UCHUN BIR MASALANING YAGONALIGI

Oltiboyeva Dilnora Orifjonovna, Madaminova Gulruh Qadamboy qizi
Jizzax davlat pedagogika universiteti, matematika va informatika fakulteti 4-bosqich talabalari

Annotasiya: Ushbu ishda Gelmgolts tenglamasi uchun masalaning yagonaligi ko'rib chiqilgan.

Kalit so'zlar: Gelmgolts tenglama, ulash sharti, Gauss-Ostrogradskiy formulasi.

Ω_0 sohada quyidagi

$$u_{xx}+u_{yy}-\lambda^2 u=0 \qquad (1)$$

Gelmgolts tenglamasini qaraymiz. Bu yerda Ω_0 - $\overline{\sigma}=\{(x,y):x^2+y^2=1,\ x\geq 0, y\geq 0\}$ chorak aylana, $\overline{OA}=\{(x,0):0\leq x\leq 1\}$ va $\overline{OB}=\{(0,y):0\leq y\leq 1\}$ kesmalar bilan chegaralangan soha, λ -sonli parametr.

$E^{(3)}$ **masala:** Shunday $u(x,y)\in C^1\left(\overline{\Omega_0}\right)\cup C^2(\Omega_0)$ funksiya topilsinki, Ω_0 sohada (1) tenglamani va quyidagi

$$u(x,y)=\varphi(x,y),(x,y)\in\sigma_0, \qquad (2)$$

$$u(z,0)=-u(0,z)+g_1(z),0\leq z\leq 1, \qquad (3)$$

$$\lim_{y\to+0}u_y(z,y)=\lim_{x\to+0}u_x(x,z)+g_2(z),0<z<1, \qquad (4)$$

shartlarni qanoatlantirsin, bu yerda $\varphi(x,y)$, $g_1(z)$, $g_2(z)$- berilgan uzluksiz funksiyalar, $u(z,0)$, $u(0,z)\in C[0,1]\cap C^2[0,1]$, $\lim\limits_{y\to+0}u_y(z,y)$, $\lim\limits_{x\to+0}u_x(x,z)\in C^2(0,1)$ bo'lib, $z\to 0, z\to 1$da 1 dan kichik maxsuslikka ega bo'lishi mumkin;

$$\varphi(1,0)+\varphi(0,1)=g_1(1).$$

$u(x,y)$ funksiya $E^{(3)}$ masalaning yechimi bo'lsin. Ushbu belgilashlarni kiritamiz:

$$\tau_1(x)=u(x,0),\ 0\leq x\leq 1;\ \nu_1(x)=\lim_{y\to+0}u_y(x,y),\ 0<x<1;$$

$$\tau_2(y)=u(0,y),\ 0\leq y\leq 1;\ \nu_2(y)=\lim_{x\to+0}u_x(x,y),\ 0<y<1;$$

Teorema. $E^{(3)}$ masalaning yechimi mavjud bo'lsa, u yagonadir.

Isbot. Faraz qilaylik, $E^{(3)}$masalaning yechimi ikkita bo'lsin. Ularning ayirmasini

$$u(x,y)=u_1(x,y)-u_2(x,y) \qquad (5)$$

deb belgilaylik. U holda $E^{(3)}$masalaga mos quyidagi bir jinsli masalaga ega bo'lamiz:

$$u_{xx}+u_{yy}-\lambda^2 u=0, \qquad (6)$$
$$u(x,y)=0,\ (x,y)\in\sigma_0;$$
$$u(z,0)=-u(0,z),\quad 0\le z\le 1;$$
$$\lim_{y\to+0}u_y(z,y)=\lim_{x\to+0}u_x(x,z),\ \ 0<z<1.$$

Agar biz yangi hosil bo'lgan bu masalaning yechimini $u(x,y)\equiv 0$ ekanligini isbotlay olsak, (5) ga asosan $E^{(3)}$masalaning yechimini yagonaligini isbotlagan bo'lamiz. Buning uchun (6) tenglamani $u(x,y)$ ga ko'paytirib, so'ngra quyidagi hisoblashlarni bajaramiz:

$$(uu_x)_x=u_xu_x+uu_{xx}=uu_{xx}+u_x^2;$$
$$(uu_y)_y=u_yu_y+uu_{yy}=uu_{yy}+u_y^2;$$

Bulardan kelib chiqadiki,

$$u(u_{xx}^2+u_{yy}^2-\lambda^2u)=(uu_x)_x+(uu_y)_y-[u_x^2+u_y^2+\lambda^2u^2].$$

Oxirgi tenglikni ikkala tomonini Ω_0 soha bo'yicha integrallaymiz va Gauss-Ostrogradskiy formulasini [1] qo'llaymiz:

$$\iint_{\Omega_0}\left\{(uu_x)_x+(uu_y)_y\right\}dxdy=\iint_{\Omega_0}[u_x^2+u_y^2+\lambda^2u^2]dxdy.$$

Bu tenglikdan quyidagi ifodani hosil qilamiz:

$$\iint_{\Omega_0}\left\{(uu_x)_x+(uu_y)_y\right\}dxdy=\int_{\partial\Omega_0}uu_xdy-uu_ydx=\int_{OAB}u(u_xdy-u_ydx)=\Big(\int_{OA}+\int_{AB}+\int_{BO}\Big)u(u_xdy-u_ydx)$$

,
$$\iint_{\partial\Omega_0}[uu_xdy-uu_ydx]=-\int_0^1\tau_2(y)\nu_2(y)dy-\int_0^1\tau_1(x)\nu_1(x)dx,$$

$$\iint_{\partial\Omega_0}[uu_xdy-uu_ydx]=-\int_0^1\tau_2(y)\nu_2(y)dy+\int_0^1\tau_2(y)\nu_2(y)dy.$$

$\iint_{\partial\Omega_0}[uu_xdy-uu_ydx]=0$ tenglikdan $\iint_{\Omega_0}\left\{u_x^2+u_y^2+\lambda^2u^2\right\}dxdy=0$ kelib chiqadi.

Oxirgi tenglikdan $u_x(x,y)\equiv 0$, $u_y(x,y)\equiv 0$, $(x,y)\in\Omega_0$, ya'ni $u(x,y)=const$ ekanligi kelib chiqadi.

Demak, $u(x,y)\in C(\overline{\Omega_0})$ bo'lgani uchun $u(x,y)\equiv 0$, $(x,y)\in\overline{\sigma}$. Bu tenglikni e'tiborga olsak, masalaning yechimini Ω_0 sohada $u(x,y)\equiv 0$ ekanligi kelib chiqadi, ya'ni masalaning yechimi mavjud bo'lsa u yagona bo'ladi.

Foydalanilgan adabiyotlar ro'yxati:

1. Г.М.Фихтенголц. Математик анализ асослари. –Тошкент.2-Т. 1972.458б
2. www.ziyouz.com