TUB SONLAR ISHTIROK ETGAN DIOFANT YAQINLASHISHLARI HAQIDA

Kattaxo‘jayeva Jahonbibi
Termiz davlat universiteti magistranti

Annotatsiya. Ushbu maqolada $p_1^c + p_2^c + p_3^c = N$ tenglamaning c soni birga yaqinlashgan holati bo’yicha diofant yaqinlashishlari masalasi qaralgan. Ushbu natija oldin olingan Tolevning ishlarida olingan natijalarni yaxshilaydi.

Kalit so’z. tub son, diofant tengsizliklar, sonning butun qismi, eksponensial yig’indi, Chebishyev funksiyasi.

1952-yilda Piatetski-Shapiro [1] Goldbax-Waring muammosiga o‘xshash quyidagi masalani ko‘rib chiqdi. $c > 1$ butun son emas deb faraz qilamiz va ε musbat son bo‘lsin. Agar r yetarlicha katta c ga bog‘liq butun son bo‘lsa, u holda

$$| p_1^c + p_2^c + \ldots + p_r^c - N | < \varepsilon \qquad (1)$$

tengsizlik yetarlicha katta N uchun $p_1, \ldots, p_r$ tub sonlarda yechimga ega bo‘ladi. Aniqroq qilib aytadigan bo'lsak, har bir $\varepsilon > 0$ uchun (1) tengsizlik tub sonlarda yechimga ega bo`ladigan eng kichik $r$ lar uchun $N > N_0(c, \varepsilon)$ bo`lib, ularning sonini $H(c)$ orqali belgilasak, u holda quyidagi limit mavjud:

$$\limsup_{c \to \infty} \frac{H(c)}{c \log c} \le 4.$$

Shuningdek, agar $1 < c < 3/2$ bo‘lganda $H(c) < 5$ bo`lishini Pyatetski-Shapiro ham isbotlagan. Agar c birga yaqinlashuvchi bo'lsa, $H(c) < 3$ deb faraz qilishimiz mumkin. Boshqa tomondan, N cheksizlikka intilganda berilgan ε o'rniga N ga qarab nolga yaqinlashuvchi bo'lgan ε ni ko'rib chiqishimiz mumkin.

Bu faraz [2] ishda $1 < c < 27/26$ va

$$\varepsilon = N^{-(1/c)(27/26 - c)} \log^{13} N$$

ekanligi isbotlangan. Ushbu maqolada oxirgi natijani yaxshilaymiz va quyidagi teoremani isbotlaymiz:

Teorema. Agar $1 < c < 9/8$ bo‘lsa, u holda shunday $N_0(c) > 0$ soni mavjudki, har qanday $N > N_0(c)$ uchun quyidagi tengsizlik p_1, p_2, p_3 tub sonlar uchun yechimga ega:

$$\left| p_1^c + p_2^c + p_3^c - N \right| < N^{-(1/c)(9/8 - c)} \log^7 N .$$

Belgilashlar

$1<c<9/8$ haqiqiy son, N yetarlicha katta son.

$$X=\left(N/2\right)^{1/c} \tag{2}$$

$$\tau = X^{7/16-c/2} \tag{3}$$

$$T = X^{23/32+c/4} \tag{4}$$

$$\varepsilon = X^{-9/8+c}\log^6 X \tag{5}$$

$$\Delta=\frac{1}{100}\varepsilon \tag{6}$$

$$r=\left[\log X\right] \tag{7}$$

bu yerda $\left[a\right]-a$ haqiqiy sonning butun qismini bildiradi;

$$K = X^{9/8-c}\left(\log X\right)^{-4} \tag{8}$$

m, n, k, l, d, r (indeks yoki indekssiz) - butun sonlar,

$p_1, p_2, p_3 \ldots$ - tub sonlar;

$\tau(n)$ - n ning musbat bo'luvchilari soni,

$\Lambda(n)$ - Mangoldt funktsiyasi,

$\Psi(n)=\sum\limits_{n\le x}\Lambda(n)$ - Chebishyev funksiyasi

$\sum\limits_{a<\gamma<b}$ - $\zeta(s)$ ning trivial bo`lmagan nollari bo'yicha yig'indisi $a<\gamma<b$, $e(x)=e^{2\pi i x}$

$$S\left(x\right)=\sum_{X/2<p\le X}\log p\; e\left(p^c x\right), \tag{9}$$

$$I\left(x\right)=\int_{X/2}^{X} e\left(t^c x\right)dt\,, \tag{10}$$

$$I_\theta\left(x\right)=\int_{X/2}^{X} e\left(t^c x\right)t^{\theta-1}dt\,, \tag{11}$$

$$J\left(x\right)=\sum_{|\gamma|\le T} I_\theta\left(x\right), \tag{12}$$

bo’lsin. O-terminlar va □ -belgilardagi o`zgarmaslar.

Teoremani isbotlash uchun bizga zarur bo`ladigan ba'zi lemmalarni keltirib o`tamiz.

1-lemma. Faraz qilaylik a, δ lar $0<\delta<a/4$ shartni qanoatlantiruvchi haqiqiy sonlar, k esa butun son bo‘lsin. U holda u k marta uzluksiz differensiallanuvchi va quyidagi shartlarni qanoatlantiruvchi $\varphi(y)$ funksiya mavjud

$$\varphi(y)=1, \qquad for \quad |y|\le a-\delta\,,$$

$$0<\varphi(y)<1, \qquad for \quad a-\delta<|y|\le a+\delta\,,$$

$$\varphi(y)=0, \qquad for \quad |y|\ge a+\delta\,,$$

uning Furye almashtirishi quyidagicha

$$\Phi(x)=\int_{-\infty}^{\infty} e(-xy)\varphi(y)dy$$

bo`lib, ushbu tengsizlikni qanoatlantiradi

$$|\Phi(x)|\le \min\left(2a, \frac{1}{\pi|x|}, \frac{1}{\pi|x|}\left(\frac{k}{2\pi|x|\delta}\right)^{k}\right).$$

2-lemma. Faraz qilaylik $G(x), F(x)$ $[a,b]$ oraliqda chegaralangan haqiqiy funksiyalar, $a\le x\le b$ uchun $G(x)\le H$ va $G(x)/F'(x)$ monoton funksiya bo`lsin va $I=\int_a^b G(x)e(F(x))dx$ ko`rinishda aniqlansin. Agar barcha $x\in[a,b]$ uchun $F'(x)\ge h>0$ yoki barcha $x\in[a,b]$ lar uchun $F'(x)\le -h<0$ bo`lsa, u holda

$$|I|\ll \frac{H}{h}$$

bo‘ladi. Agar barcha $x\in[a,b]$lar uchun $F''(x)\ge h>0$ bo‘lsa, u holda

$$|I|\ll \frac{H}{\sqrt{h}}$$

o`rinli bo`ladi.

3-lemma. Ushbu baholar o`rinli:

$$\int_{-\tau}^{\tau}\left|S^2(x)\right|dx \ll X^{2-c}\log^3 X\,, \tag{i}$$

$$\int_{-\tau}^{\tau}\left|I^2(x)\right|dx \ll X^{2-c}\log X\,, \tag{ii}$$

$$\int_{n}^{n+1}\left|S^{2}(x)\right|dx \ll X\log^{3}X \qquad \text{(iii)}$$

n ga nisbatan bir xil.

4-lemma. Agar $\tau \le |x| \le K$ bo'lsa, u holda

$$\left|S(x)\right| \ll X^{7/8}\log^{3}X.$$

baho o`rinli bo`ladi.

1-4-lemmalarning isboti D. I. Tolev [3] ishidagi 1, 2, 7, 10-lemmalar kabi isbotlanadi.

Teoremaning isboti.

$$B=\sum_{\substack{X/2<p_1,p_2,p_3\le X\\ \left|p_1^c+p_2^c+p_3^c-N\right|<\varepsilon}}\log p_1\log p_2\log p_3$$

yig‘indisi X cheksizlikka intilganda yig`indi ham cheksizlikka intilishini ko‘rsatish kifoya.

$$B_1=\sum_{X/2<p_1,p_2,p_3\le X}\log p_1\log p_2\log p_3\varphi(p_1^c+p_2^c+p_3^c-N)$$

hisoblaymiz. $\varphi(y)$ ning ta’rifi bo‘yicha biz

$$B\ge B_1 \qquad (13)$$

ga ega bo'lamiz. Furye almashtirish formulasidan bizga

$$B_1=\sum_{X/2<p_1,p_2,p_3\le X}\log p_1\log p_2\log p_3\int_{-\infty}^{\infty}e((p_1^c+p_2^c+p_3^c-N)x)\Phi(x)dx=$$

$$=\int_{-\infty}^{\infty}S^{3}(x)e(-Nx)\Phi(x)dx$$

ga ega bo‘lamiz. B_1 ni

$$B_1=D_1+D_2+D_3, \qquad (14)$$

ko‘rinishda ifodalaylik, bu yerda

$$D_1=\int_{-\tau}^{\tau}S^{3}(x)e(-Nx)\Phi(x)dx,$$

$$D_2=\int_{\tau<|x|<K}S^{3}(x)e(-Nx)\Phi(x)dx,$$

$$D_3 = \int_{|x|>K} S^3(x)e(-Nx)\Phi(x)dx\,.$$

(5)-(8) va 1-lemmadan foydalanib, biz

$$|D_3| \ll \int_K^\infty |S^3(x)||\Phi(x)|dx \ll X^3\int_K^\infty \frac{1}{x}\left(\frac{r}{2\pi\Delta x}\right)^r dx \ll X^3\left(\frac{r}{2\pi\Delta K}\right)^r \ll 1. \qquad (15)$$

ga egamiz.

Endi D_2 ni ko'rib chiqamiz:

$$|D_2| \ll \int_\tau^K |S^3(x)||\Phi(x)|dx \ll \left(\max_{\tau\le x\le K}|S(x)|\right)\int_\tau^K |S^2(x)||\Phi(x)|dx\,. \qquad (16)$$

1-lemmaga binoan

$$\int_\tau^K |S^2(x)||\Phi(x)|dx \ll \varepsilon\int_\tau^{1/\varepsilon}|S^2(x)|dx + \int_{1/\varepsilon}^K |S^2(x)|\frac{dx}{x} \ll$$

$$\ll \varepsilon\sum_{0\le n\le 1/\varepsilon}\int_n^{n+1}|S^2(x)|dx + \sum_{1/\varepsilon-1\le n\le K}\frac{1}{n}\int_n^{n+1}|S^2(x)|dx$$

olamiz. Oxirgi baho va 3-lemma (iii)

$$\int_\tau^K |S^2(x)||\Phi(x)|dx \ll X\log^4 X$$

ni beradi. Shuning uchun (16) va 4-lemma orqali biz

$$|D_2| \ll \varepsilon\frac{X^{3-c}}{\log X} \qquad (17)$$

ga egamiz.

Endi biz D_1 uchun asimptotik formulani topamiz.

$$H_1 = \int_{-\tau}^{\tau} I^3(x)e(-Nx)\Phi(x)dx$$

Belgilaymiz. Lemmalar 3 (i), 3 (ii) dan foydalanib, biz

$$|D_1 - H_1| \ll \int_{-\tau}^{\tau}|S^3(x)-I^3(x)||\Phi(x)|dx \ll \varepsilon\int_{-\tau}^{\tau}|S(x)-I(x)||S^2(x)+I^2(x)|dx \ll$$

$$\ll \varepsilon X e^{-(\log X)^{1/5}}\left(\int_{-\tau}^{\tau}|S^2(x)|dx + \int_{-\tau}^{\tau}|I^2(x)|dx\right) \ll \varepsilon X^{3-c}e^{-(\log X)^{1/6}} \qquad (18)$$

ga egamiz.

$$H = \int_{-\infty}^{\infty} I^3(x)e(-Nx)\Phi(x)dx$$

belgilaymiz. U holda bizda

$$|H - H_1| \ll \int_{\tau}^{\infty} |I^3(x)||\Phi(x)| dx \qquad (19)$$

ga ega bo`lamiz. 2-lemma orqali biz $|I(x)| \ll 1/\left(|x| X^{c-1}\right)$ ni olamiz. Shuning uchun (19) va 1-lemma orqali biz

$$|H - H_1| \ll \frac{1}{X^{3(c-1)}} \int_{\tau}^{\infty} |\Phi(x)| \frac{dx}{x^3} \ll \frac{\varepsilon}{X^{3(c-1)}\tau^2} \ll \varepsilon \frac{X^{3-c}}{\log X} \qquad (20)$$

olamiz.

Endi (14), (15), (17) - (20) formulalardan

$$B_1 = H + O\left(\frac{\varepsilon X^{3-c}}{\log X}\right)$$

ekanligi kelib chiqadi. Demak, bizda $B_1 \gg \varepsilon X^{3-c}$ mavjud. Bu (14) bilan birgalikda $B \gg \varepsilon X^{3-c}$ va shuning uchun $B \to \infty$ da $X \to \infty$ ekanligini anglatadi. Teorema isbotlandi.

Foydalanilgan adabiyotlar

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