ALGEBRAIK MISOLLARNI GEOMETRIK USULLARDA YECHIMI

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Bu maqolada algebraik misollar bilan bog’liq masalalar keltirilgan. Bu masalalar iqtidorli o’quvchi, talabalarning matematika faniga bo’lgan qiziqishlarini orttirishga, mustaqil ravishda bilim saviyalarini oshirishga va mantiqiy fikrlashga yordam beradi. Maqolada algebra va geometriya fanlarining integratsiyasini ko’rish mumkin**.**

Tayanch so’zlar. Teskari trigonometrik funksiyalar, burchaklar,to’g’ri burchakli uchburchak, uchburchak balandligi, cosinuslar teoremasi.algebraik misollar.

Keywords: Angles, right triangle,triangle height, cosine theorem,algebraic examples.

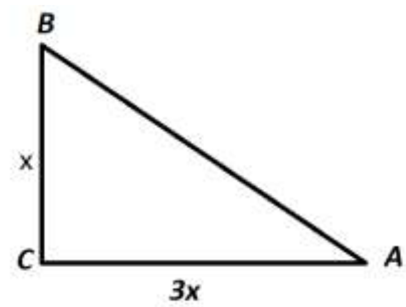
Ключевые слова: Обратные тригонометрические функции, глы, прямоугольный треугольник, высота треугольника, теорема косинусов,алгебраические примеры.

Bizga quyidagicha misollar berilgan bo’lsin:

1-Misol. Hisoblang.

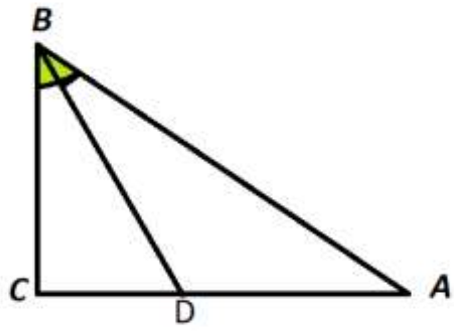
I = arctg3-arcsin $\frac{\sqrt{5}}{5}$

Yechish. Bu trigonometrik ifodani hisoblashda to’g’ri burchakli ABC uchburchakdan foydalanamiz. <ABC=argtg3 tenglikka ega bo’lamiz.(1-rasm).

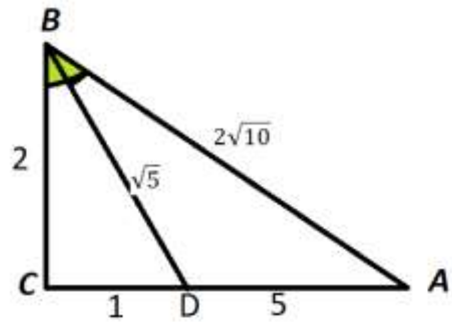


1-rasm

Umumiy holda <ABC - <DBC = <ABD tenglikdan <DBC = arcsin $\frac{\sqrt{5}}{5}$ va I ayirma esa < ABD ga teng ekanligi ko’rish mumkin. (2-rasm).



2-rasm



3-rasm

BDC uchburchakda $<DBC = \arcsin\frac{\sqrt{5}}{5}$ ekanligidan DC = 1 va BD = $\sqrt{5}$.

Pifagor teoremasiga ko’ra $BC = \sqrt{BD^2 - DC^2} = 2$ 2-rasmdagi chizmaga ko’ra BC=2, demak AC=6.

ABC uchburchak uchun pifagor teoremasini qo’llab: $AB = 2\sqrt{10}$ (3-rasm). ABD uchburchak uchun kosinuslar teoreamasini qo’llab so’ralayotgan I=<ABD ni topamiz.

$$AD^2 = AB^2 + BD^2 - 2\cdot AB\cdot BD\cos < ABD,$$

$5^2 = (2\sqrt{10})^2 + (\sqrt{5})^2 - 2\cdot 2\sqrt{10}\cdot\sqrt{5}\cdot\cos < ABD$, natija $< ABD = \frac{\pi}{4}$

Javob: I = arctg3-arcsin $\frac{\sqrt{5}}{5} = \frac{\pi}{4}$

2-Misol. Quyidagi tenglama yechimini toping: Bu yerda $0 < x < \frac{\pi}{2}$

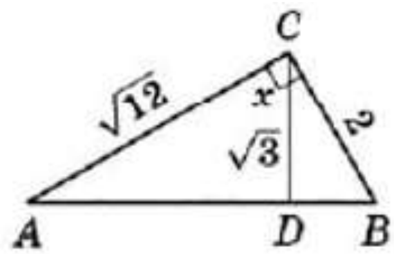
$$\sqrt{15 - 12\cos x} + \sqrt{7 - 4\sqrt{3}\sin x} = 4$$

Yechim: Tenglamani quyidagi ko’rinishda yozib olamiz:

$$\sqrt{(\sqrt{12})^2 + (\sqrt{3})^2 - 2\cdot\sqrt{12}\cdot\sqrt{3}\cdot\cos x} + \sqrt{2^2 + (\sqrt{3})^2 - 2\cdot 2\cdot\sqrt{3}\cos(\frac{\pi}{2} - x)} = 4$$

2 ta uchburchakni umumiy tomoni CD= $\sqrt{3}$ ko’rib chiqamiz. ACD uchburchakda AC= $\sqrt{12}$, <ACD=x. BCD uchburchakda esa BC = 2, <BCD = $\frac{\pi}{2} - x$.

A va B nuqtalar rasmdagi ko'rinishda turibdi.Chap tomondagi tenglik AD+BD=4 hosil bo'ladi. Shunday qilib to'g'ri burchakli ACB uchburchakda AB gipotenuzasi bo'ladi. $\sqrt{AC^2+CB^2}=4$ hosil bo'ladi. CD kesmaning D uchi AB tomonga joylashgan. Bundan $CD=\dfrac{AC\cdot BC}{AB}=\sqrt{3}$ quyidagi tenglik hosil qilamiz. Demak CD kesma uchburchak ABC ning balandligi.



Natijada $\cos x=\dfrac{CD}{AC}=\dfrac{\sqrt{3}}{\sqrt{12}}=\dfrac{1}{2}$ tenglikdan

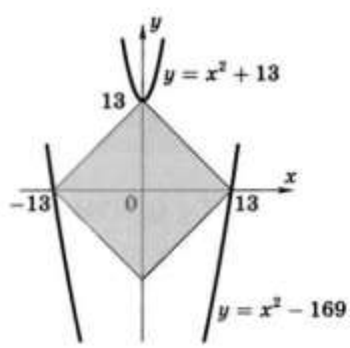
$$x=\frac{\pi}{3}$$

Javob: $x=\dfrac{\pi}{3}$.

3-Misol. Agar $|x|+|y|\le 13$ bo'lsa, $y-x^2$ ifodaning eng katta va eng kichik qiymatini toping.

Yechim: $|x|+|y|=13$ tenglamaning grafigi markazi koordinatalar boshida bo'lgan kvadratni aniqlaydi. Shuning uchun bu tengsizlik ushbu kvadratga tegishli barcha nuqtalarning koordinatalarini qanoatlantiradi.(5-rasm).

$y-x^2=t$ belgilashni kiritsak, yuqoridagi ifodaning t o'zgaruvchiga nisbatan eng kata va eng kichik qiymatini aniqlashga kelamiz. $y=x^2+t$ funksiya va kvadrat umumiy nuqtalarga ega bo'ladi.



5-rasm

Berilgan funksiyaning grafigini $y=x^2$ funksiyaning grafigini y o'qi bo'yicha parallel ko'chirish orqali tuziladi. Bundan esa:

1) Agar $y=x^2+t$ parabola (0;13) nuqtadan o'tsa, t o'zgaruvchi o'zining eng kata qiymatiga ega bo'ladi, bunda t=13 ga teng qiymat qabul qiladi;

2) $y=x^2+t$ parabola (13;0) va (-13;0) nuqtalardan o'tsa, t o'zgaruvchi o'zining eng kichik qiymatiga ega bo'ladi,bunda t=-169 ga teng qiymat qabul qiladi.

Javob:Eng katta qiymat 13 ga teng, ng kichik qiymat -169 ga teng.

4-misol. x,y va z musbat sonlar uchun $\begin{cases} x^2+xy+\frac{1}{3}y^2=25 \\ \frac{1}{3}y^2+z^2=9 \\ z^2+zx+x^2=16 \end{cases}$

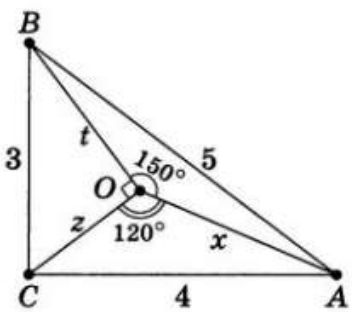
Tenglamalar sistemasidan xy+2yz+3zx ifodaning qiymatini toping.

Yechilishi: $t=\frac{y}{\sqrt{3}}$ belgilashni kiritib

$$\begin{cases} x^2+xt\sqrt{3}+t^2=25 \\ t^2+z^2=9 \\ z^2+zx+x^2=16 \end{cases} \quad \text{bundan} \quad \begin{cases} x^2+2xt\cos150^0+t^2=5^2 \\ t^2+z^2=9 \\ z^2-2zx\cos120^0+x^2=4^2 \end{cases}$$

sistemani hosil qilamiz.

Tekislikda O nuqtani va $\angle BOC=90^0$,$\angle AOC=120^0$,$\angle AOB=150^0$ shartalarni qanoatlantiruvchi OA=x, OB=t va OC=z kesmalarni belgilab olamiz. Bundan esa $90^0+120^0+150^0=360^0$ tenglik hosil bo'ladi.



U holda 2 sistemadan BC=3 (Pifagor teoremasiga ko'ra) 3 va 1 tenglamadan AC=4, AB=5 (cosinuslar teoremasiga ko'ra) ga ega bo'lamiz. Natijada ABC to'g'ri burchakli uchburchak hosil bo'ladi.

Qaralayotgan ifodaning ko'rinishi:

$$xy+2yz+3zx=xt\sqrt{3}+2tz\sqrt{3}+3zx=4\sqrt{3}\left(\frac{1}{2}xt\cdot\sin150^0+\frac{1}{2}tz+\frac{1}{2}zx\cdot\sin120^0\right)=$$

$$4\sqrt{3}(S_{\Delta AOB}+S_{\Delta BOC}+S_{\Delta COA})=4\sqrt{3}\cdot S_{\Delta ABC}=2\sqrt{3}\cdot AC\cdot BC=24\sqrt{3}.$$

Javob: $24\sqrt{3}.$

Foydalanilgan adabiyotlar. 11-sinf algebra geometriya,Geometrik va geometrik bo'lmagan topshiriqlar(A.D. BLINKOV)